Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Formal Specification and Verification"
Exercise sheet 1

Exercise 1.1:
Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

(1) $(P \wedge \neg Q)$
(2) $(R \vee \neg S)$
(3) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$
(4) Q
(5) S
(6) V

Exercise 1.2:
Let F be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge (Q \rightarrow P) \wedge \neg P)] \wedge (P \vee R)$$

(1) Compute the negation normal form (NNF) F' of F.
(2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 1.3:
Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F.

Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Exercise 1.4:
Consider the following deductive system for propositional logic (with signature $\neg, \rightarrow$):

Axiom schemata:

(1) $\neg p \rightarrow (p \rightarrow q)$

(2) $p \rightarrow (q \rightarrow p)$

(3) $(p \rightarrow q) \rightarrow ((\neg p \rightarrow q) \rightarrow q)$

(4) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

Inference rules

Modus Ponens: $\frac{p, \quad p \rightarrow q}{q}$

(1) Show that for all formulae F and G, $\{F, \neg F\} \vdash G$ in this system.

(2) Give a proof for $F \to F$ in this system.

Hint: For (2) you can e.g. use instances of axiom schema 2 (twice), 4, and Modus Ponens (twice).

Supplementary exercise (to be discussed on May 3, 2012)

Let F be a formula containing neither $\to$ nor $\leftrightarrow$, P a propositional variable not occurring in F, and F' a subformula of F.

Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \to F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \to P)$ is satisfiable.

Please submit your solution until Wednesday, May 2, 2012 at 11:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSW" in the subject.
- Hand it in to me (Room B 225) or to Markus Bender (Room B 224).