Universität Koblenz-Landau

FB 4 Informatik

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<div align="center">

Exercises for "Formal Specification and Verification"
Exercise sheet 2

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Exercise 2.1:
Give a proof for

$$\Rightarrow ((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$$

in the sequent calculus for propositional logic presented in the lecture.

Exercise 2.2:
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

$$
\begin{array}{cc}
(1) & \neg P \vee \neg Q \vee R \\
(2) & \neg P \vee \neg Q \vee S \\
(3) & P \\
(4) & \neg S \vee \neg R \\
(5) & Q
\end{array}
$$

Exercise 2.3:
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$

(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Exercise 2.4:
Consider the following boolean formula $F := (P \wedge ((Q \wedge \neg R) \vee (\neg Q \wedge R))) \vee (\neg P \wedge \neg R)$.

Construct a reduced BDD for F such that the root is a P-node followed by Q- and then R-nodes.

Supplementary exercise
(to practice optimized structure-preserving translation to clause form and checking unsatisfiability with different methods)

Exercise 2.5:
Let F be the following formula:

$$\neg((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$$

(1) Compute the negation normal form (NNF) F' of F.

(2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

(3) Let F'' be the CNF obtained this way. Prove that F' is unsatisfiable using the following methods:

- resolution
- DPLL

(4) Does it follow that F is unsatisfiable? Justify your answer.

Please submit your solution until Wednesday, May 23, 2012 at 11:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSW" in the subject.

- Hand it in to me (Room B225) or drop it in the box in front of Room B224.