Universität Koblenz-Landau

FB 4 Informatik

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Exercises for “Formal Specification and Verification”
Exercise sheet 3

Exercise 3.1:
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae, which are neither?

(a) $\neg p(g(a), f(x, y))$

(b) $f(x, x) \approx x$

(c) $p(f(x, a), x) \vee p(a, b)$

(d) $p(\neg g(x), g(y))$

(e) $\neg p(f(x, y))$

(f) $p(a, b) \wedge p(x, y) \wedge y$

(g) $\exists y(\neg p(f(y, y), y))$

(h) $\forall x \forall y(g(p(x, y)) \approx g(x))$

Exercise 3.2:
Compute the results of the following substitutions:

(a) $f(g(x), x)[g(a)/x]$

(b) $p(f(y, x), g(x))[x/y]$

(c) $\forall y(p(f(y, x), g(y)))[x/y]$

(d) $\forall y(p(f(y, x), x))[y/x]$

(e) $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$

(f) $\exists y\big(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x)\big)[g(y)/y, g(z)/x]$

Exercise 3.3:
Prove or refute the following statements:

(a) If F is a first-order formula, then F is valid if and only if $F \rightarrow \bot$ is unsatisfiable.

(b) If F and G are first-order formulae, F is valid, and $F \rightarrow G$ is valid, then G is valid.

(c) If F and G are first-order formulae, F is satisfiable, and $F \rightarrow G$ is satisfiable, then G is satisfiable.

(d) If F is a first-order formula and x a variable, then F is unsatisfiable if and only if $\exists x F$ is unsatisfiable.

(e) If F and G are first-order formulae and x is a variable then $\forall x(F \wedge G) \models \forall x F \wedge \forall x G$ and $\forall x F \wedge \forall x G \models \forall x(F \wedge G)$.

(f) If F and G are first-order formulae and x is a variable then $\exists x(F \wedge G) \models \exists x F \wedge \exists x G$ and $\exists x F \wedge \exists x G \models \exists x(F \wedge G)$.

Exercise 3.4:
Let $\Sigma = (\Omega, \Pi)$, where $\Omega = \{0/0, s/1, +/2\}$ and $\Pi = \emptyset$ (i.e. the only predicate symbol is $\approx$). Consider the following formulae in the signature Σ:

1. $F_1 = \forall x \ (x + 0 \approx x)$
2. $F_2 = \forall x, y \ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y \ \ (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

Please submit your solution until Wednesday, May 23, 2012 at 11:00.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSW" in the subject.
- Hand it in to me (Room B225) or drop it in the box in front of Room B224.