Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Formal Specification and Verification"
Exercise sheet 9

Exercise 9.1:
Show that the following formulae are valid in propositional dynamic logic (i.e. true in all PDL Kripke models) :

(1) $[\alpha](A \wedge B) \leftrightarrow [\alpha]A \wedge [\alpha]B$
(2) $[\alpha;\beta]A \leftrightarrow [\alpha][\beta]A$
(3) $[\alpha \cup \beta]A \leftrightarrow [\alpha]A \wedge [\beta]A$
(4) $[A?]B \leftrightarrow (A \rightarrow B)$
(5) $[\alpha^*]A \leftrightarrow A \wedge [\alpha][\alpha^*]A$
(6) $[\alpha^*](A \rightarrow [\alpha]A) \rightarrow (A \rightarrow [\alpha^*]A)$

Exercise 9.2:
Consider the axiom system for PDL presented in the lecture:

Axiom schemes

$(D1)$	All propositional logic tautologies
$(D2)$	$[\alpha](A \rightarrow B) \rightarrow ([\alpha]A \rightarrow [\alpha]B)$
$(D3)$	$[\alpha](A \wedge B) \leftrightarrow [\alpha]A \wedge [\alpha]B$
$(D4)$	$[\alpha;\beta]A \leftrightarrow [\alpha][\beta]A$
$(D5)$	$[\alpha \cup \beta]A \leftrightarrow [\alpha]A \wedge [\beta]A$
$(D6)$	$[A?]B \leftrightarrow (A \rightarrow B)$
$(D7)$	$[\alpha^*]A \leftrightarrow A \wedge [\alpha][\alpha^*]A,$
$(D8)$	$[\alpha^*](A \rightarrow [\alpha]A) \rightarrow (A \rightarrow [\alpha^*]A)$

Inference rules

MP	$\frac{P, \quad P \rightarrow Q}{Q}$
Gen	$\frac{F}{[\alpha]F}$

Show that for every $n \in \mathbb{N}$, $n \geq 1$, the following formulae are provable in this axiom system:

(1) $[\alpha^n]F \leftrightarrow [\alpha]^n F$ (2) $[\alpha^*]F \rightarrow [\alpha]^n F$

where $[\alpha^0]F = F$; $\alpha^n = \underbrace{\alpha;\ldots;\alpha}_{n \text{ times}}$ and $[\alpha]^0 F = F$; $[\alpha]^n F = \underbrace{[\alpha]\ldots[\alpha]}_{n \text{ times}} F$

Please submit your solution until Wednesday, July 11, 2012 at 11:00. Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSV" in the subject.
- Hand it in to me (Room B225) or drop it in the box in front of Room B224.