

Lemma  $\text{Sat}(E(F \cup G))$  smallest  $\text{nt}$  with

(1)  $\text{Sat}(G) \subseteq T$

(2)  $\{s \in \text{Sat}(F) \mid \text{Post}(s) \cap T \neq \emptyset\} \subseteq T$   
(=)

(A) Show  $T = \text{Sat}(E(F \cup G))$  w.r.t. (1) and (2)

Follows from the fact that

$$E(F \cup G) = G \vee (F \wedge E \circ E(F \cup G))$$

(1)  $\text{Sat}(G) \subseteq \text{Sat}(E(F \cup G))$

(2)  $s \in \text{Sat}(F) \ \& \ \exists t \in S: s \rightarrow t \ \& \ t \in E(F \cup G) \Rightarrow \begin{cases} s \in \text{Sat}(F) \\ s \in \text{Sat}(E \circ E(F \cup G)) \end{cases}$

(B) For any  $T$  satisfying (1) and (2)

$$\text{Sat}(E(F \cup G)) \subseteq T \quad (\text{smallest set with this property})$$

Let  $s \in \text{Sat}(E(F \cup G))$

Case 1:  $s \in \text{Sat}(G)$  then  $s \in T$

Case 2:  $s \notin \text{Sat}(G)$

Then  $\exists$  path  $\pi = s_0 \dots s_n \dots$

$$\text{nt. } \pi \neq \emptyset \cup \Psi \quad \parallel \quad \text{S}$$

Let  $n \geq 0$  nt.  $s_i \neq F \quad 0 \leq i \leq n$

$$s_{n+1} \neq \emptyset$$

Then:  $s_n \in \text{Sat}(G) \subseteq T$

$s_{n+1} \in T$  since  $s_n \in \text{Post}(s) \ \& \ s_n \in T$   
&  $s_{n+1} \in \text{Sat}(F)$

$\vdots$

$s_0 \in T$  since  $s_n \in \text{Post}(s_0) \ \& \ s_1 \in T$   
&  $s_0 \in \text{Sat}(F)$ .

$\parallel$   
S.

We thus showed that also in this case  $s \in T$