

### Exercises for “Formal Specification and Verification” Exercise sheet 3

#### Exercise 3.1:

Consider the following boolean formula  $F := (P \wedge ((Q \wedge \neg R) \vee (\neg Q \wedge R)))$ .

- (1) Construct a reduced OBDD  $B_F$  for  $F$  with the order  $[P, Q, R]$  i.e. such that the root is a  $P$ -node followed by  $Q$ - and then  $R$ -nodes.
- (2) Let  $B_F$  be the OBDD constructed previously. Construct the following OBDDs:
  - (a)  $\text{restrict}(0, R, B_F)$ ;
  - (b)  $\text{restrict}(1, R, B_F)$ ;
  - (c)  $\text{exists}(R, B_F)$ .

#### Exercise 3.2:

Let  $\Sigma = (\Omega, \Pi)$  be a signature, where  $\Omega = \{f/2, g/1, a/0, b/0\}$  and  $\Pi = \{p/2\}$ ; let  $X$  be the set of variables  $\{x, y, z\}$ . Which of the following expressions are terms over  $\Sigma$  and  $X$ , which are atoms/literals/clauses/formulae, which are neither?

- (a)  $\neg p(g(a), f(x, y))$
- (b)  $f(x, x) \approx x$
- (c)  $p(f(x, a), x) \vee p(a, b)$
- (d)  $p(\neg g(x), g(y))$
- (e)  $\neg p(f(x, y))$
- (f)  $p(a, b) \wedge p(x, y) \wedge y$
- (g)  $\exists y(\neg p(f(y, y), y))$
- (h)  $\forall x \forall y(g(p(x, y)) \approx g(x))$

#### Exercise 3.3:

Let  $\Sigma = (S, \Omega, \Pi)$  be a many-sorted signature, where  $S = \{\text{int}, \text{list}\}$ ,  $\Omega = \{\text{cons}, \text{car}, \text{cdr}, \text{nil}, b\}$  and  $\Pi = \{p\}$  with the following arities:

$a(\text{cons}) = \text{int, list} \rightarrow \text{list}$      $a(\text{car}) = \text{list} \rightarrow \text{int}$      $a(\text{cdr}) = \text{list} \rightarrow \text{list}$   
 $a(\text{nil}) = \rightarrow \text{list}$     (i.e.  $\text{nil}$  is a constant of sort  $\text{list}$ )  
 $a(b) = \rightarrow \text{int}$     (i.e.  $b$  is a constant of sort  $\text{int}$ )  
 $a(p) = \text{int, list}$ .

Let  $X_{\text{int}}$  be the set of variables of sort  $\text{int}$  containing  $\{i, j, k\}$ , and let  $X_{\text{list}}$  be the set of variables of sort  $\text{list}$  containing  $\{x, y, z\}$ . Let  $X = \{X_{\text{int}}, X_{\text{list}}\}$ . Which of the following expressions are terms over  $\Sigma$  and  $X$ , which are atoms/literals/clauses/formulae<sup>1</sup>, which are neither?

- (a)  $\text{cons}(\text{cons}(b, \text{nil}), \text{nil})$
- (b)  $\text{cons}(b, \text{cons}(b, \text{nil}))$
- (c)  $\neg p(b, \text{cons}(b, \text{cons}(b, \text{nil})))$
- (d)  $\neg p(\text{cons}(b, \text{nil}), \text{cons}(b, \text{cons}(b, \text{nil})))$
- (e)  $\text{cons}(b, \text{cons}(b, \text{nil})) \approx_l \text{cons}(\text{cons}(x, b), \text{nil})$
- (f)  $\text{cons}(i, \text{cons}(b, \text{nil})) \approx j$
- (g)  $p(\neg \text{car}(x), x)$
- (h)  $\neg p(\text{car}(x), x) \vee p(j, \text{cons}(j, x))$
- (i)  $\neg p(b, x) \vee p(b, \text{cons}(b, x)) \vee b$
- (j)  $\forall i : \text{int}, \forall x : \text{list} (\text{cons}(\text{car}(x), \text{cdr}(x)) \approx_l x)$
- (k)  $\exists i : \text{int}, \forall y : \text{list} (\text{cons}(b, p(x, y)) \approx_l \text{cdr}(y))$

### Exercise 3.4:

Compute the results of the following substitutions:

- (a)  $f(g(x), x)[g(a)/x]$
- (b)  $p(f(y, x), g(x))[x/y]$
- (c)  $\forall y(p(f(y, x), g(y)))[x/y]$
- (d)  $\forall y(p(f(y, x), x))[y/x]$
- (e)  $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$
- (f)  $\exists y(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x))[g(y)/y, g(z)/x]$

Please submit your solution until Monday, May 6, 2013 at 18:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword “Homework FSV” in the subject.
- Hand it in to me (Room B225) or drop it in the box in front of Room B224.

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<sup>1</sup>In first-order logic with equality, where equality between terms of sort  $\text{int}$  is  $\approx_i$  and equality between terms of sort  $\text{list}$  is  $\approx_l$ .