Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **April 29, 2014**

Exercises for "Formal Specification and Verification"
Exercise sheet 1

Exercise 1.1:
Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

(1) $(P \wedge \neg Q)$	(4) Q
(2) $(R \vee \neg S)$	(5) S
(3) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$	(6) V

Exercise 1.2:
Let F be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge (Q \rightarrow P) \wedge \neg P)] \wedge (P \vee R)$$

(1) Compute the negation normal form (NNF) F' of F.

(2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 1.3:
Consider the following deductive system for propositional logic (with signature $\neg, \rightarrow$):

Axiom schemata:

(1) $\neg p \rightarrow (p \rightarrow q)$

(2) $p \rightarrow (q \rightarrow p)$

(3) $(p \rightarrow q) \rightarrow ((\neg p \rightarrow q) \rightarrow q)$

(4) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

Inference rules

Modus Ponens: $\frac{p, \quad p \rightarrow q}{q}$

Give a proof for $F \rightarrow F$ in this system.

Hint: You can e.g. use instances of axiom schema 2 (twice), 4, and Modus Ponens (twice).

Exercise 1.4:
Give a proof for

$$\Rightarrow ((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$$

in the sequent calculus for propositional logic presented in the lecture.

Exercise 1.5:
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

(1)	$\neg P \vee \neg Q \vee R$
(2)	$\neg P \vee \neg Q \vee S$
(3)	P
(4)	$\neg S \vee \neg R$
(5)	Q

Exercise 1.6:
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$
(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Supplementary exercises (to be discussed on May 8, 2014)

Exercise 1.7:
Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F.

Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Exercise 1.8:
Let F be a formula containing neither $\rightarrow$ nor $\leftrightarrow$, P a propositional variable not occurring in F, and F' a subformula of F.

Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Please submit your solution until Wednesday, May 7, 2014 at 11:00. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework FSW” in the subject.
- Put it in the box in Room B 222.