Universität Koblenz-Landau

FB 4 Informatik

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May 7, 2014

Exercises for "Formal Specification and Verification"
Exercise sheet 2

Exercise 2.1:
Let F be the following formula:

$$\neg[\ \neg(P \vee (\neg Q \vee R))\ \vee\ (\neg(P \vee Q) \vee (P \vee R))\ ]$$

Convert F to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 2.2:
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $\neg P_6 \wedge (P_2 \vee P_6) \wedge (\neg P_5 \vee P_6) \wedge (\neg P_2 \vee P \vee P_1) \wedge (\neg P_1 \vee \neg Q \vee R) \wedge (P_3 \vee P_5) \wedge (\neg P_4 \vee P_5) \wedge (\neg P_3 \vee P \vee Q) \wedge (\neg P \vee P_4)$

(2) $(U \vee \neg P_6) \wedge (\neg U \vee \neg P_6) \wedge (P_2 \vee P_6) \wedge (\neg P_5 \vee P_6) \wedge (\neg P_2 \vee P \vee P_1) \wedge (\neg P_1 \vee \neg Q \vee R) \wedge (P_3 \vee P_5) \wedge (\neg P_4 \vee P_5) \wedge (\neg P_3 \vee P \vee Q) \wedge (\neg P \vee P_4) \wedge (\neg R \vee P_4)$

Hint: (1) should be satisfiable; (2) should be unsatisfiable.

Exercise 2.3:
Consider the following boolean formula $F := (P \wedge ((Q \wedge \neg R) \vee (\neg Q \wedge R)))$.

(1) Construct a reduced OBDD B_F for F with the order $[P, Q, R]$ i.e. such that the root is a P-node followed by Q- and then R-nodes.

(2) Let B_F be the OBDD constructed previously. Construct the following OBDDs:

  (a) $\mathsf{restrict}(0, R, B_F)$;
  (b) $\mathsf{restrict}(1, R, B_F)$;
  (c) $\mathsf{exists}(R, B_F)$.

Exercise 2.4:
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae, which are neither?

(a) $\neg p(g(a), f(x, y))$
(b) $f(x, x) \approx x$
(c) $p(f(x, a), x) \vee p(a, b)$
(d) $p(\neg g(x), g(y))$
(e) $\neg p(f(x, y))$
(f) $p(a, b) \wedge p(x, y) \wedge y$
(g) $\exists y(\neg p(f(y, y), y))$
(h) $\forall x \forall y(g(p(x, y)) \approx g(x))$

Exercise 2.5:
Let $\Sigma = (S, \Omega, \Pi)$ be a many-sorted signature, where $S = \{\mathsf{int}, \mathsf{list}\}$, $\Omega = \{\mathsf{cons}, \mathsf{car}, \mathsf{cdr}, \mathsf{nil}, b\}$ and $\Pi = \{p\}$ with the following arities:

$a(\mathsf{cons}) = \mathsf{int}, \mathsf{list} \to \mathsf{list}$ $\quad a(\mathsf{car}) = \mathsf{list} \to \mathsf{int}$ $\quad a(\mathsf{cdr}) = \mathsf{list} \to \mathsf{list}$
$a(\mathsf{nil}) = \to \mathsf{list}$ (i.e. nil is a constant of sort list)
$a(b) = \to \mathsf{int}$ (i.e. b is a constant of sort int)
$a(p) = \mathsf{int}, \mathsf{list}$.

Let X_{int} be the set of variables of sort int containing $\{i, j, k\}$, and let X_{list} be the set of variables of sort list containing $\{x, y, z\}$. Let $X = \{X_{\mathsf{int}}, X_{\mathsf{list}}\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae[1], which are neither?

(a) $\mathsf{cons}(\mathsf{cons}(b, \mathsf{nil}), \mathsf{nil})$
(b) $\mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil}))$
(c) $\neg p(b, \mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})))$
(d) $\neg p(\mathsf{cons}(b, \mathsf{nil}), \mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})))$
(e) $\mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})) \approx_l \mathsf{cons}(\mathsf{cons}(x, b), \mathsf{nil})$
(f) $\mathsf{cons}(i, \mathsf{cons}(b, \mathsf{nil})) \approx j$
(g) $p(\neg \mathsf{car}(x), x)$
(h) $\neg p(\mathsf{car}(x), x) \vee p(j, \mathsf{cons}(j, x))$
(i) $\neg p(b, x) \vee p(b, \mathsf{cons}(b, x)) \vee b$
(j) $\forall i : \mathsf{int}, \forall x : \mathsf{list}\ (\mathsf{cons}(\mathsf{car}(x), \mathsf{cdr}(x)) \approx_l x)$
(k) $\exists i : \mathsf{int}, \forall y : \mathsf{list}\ (\mathsf{cons}(b, p(x, y)) \approx_l \mathsf{cdr}(y))$

Please submit your solution until Wednesday, May 14, 2014 at 11:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSV" in the subject.
- Hand it in to me (Room B225) or drop it in the box in front of Room B224.

[1] In first-order logic with equality, where equality between terms of sort int is $\approx_i$ and equality between terms of sort list is $\approx_l$.