

- 1: if $(y > 0)$ then skip else halt;
- 2: $x := 2y$;
- 3: $z := y$;
- 4: if $(x \leq z)$ then goto 6;
- 5: exit;
- 6: error

Solution to exercise 10.1

(1) $I_{init} = (pc = l_1)$; $P_{err} = (pc = l_6)$

- $P_1 = \text{move}(l_1, l_2) \wedge y > 0 \wedge \text{skip}(x, y, z)$
- $P_2 = \text{move}(l_2, l_3) \wedge x' = 2y \wedge \text{skip}(y, z)$
- $P_3 = \text{move}(l_3, l_4) \wedge z' = y \wedge \text{skip}(x, y)$
- $P_4 = \text{move}(l_4, l_5) \wedge x > z \wedge \text{skip}(x, y, z)$
- $P_5 = \text{move}(l_4, l_6) \wedge x \leq z \wedge \text{skip}(x, y, z)$

$P_R = P_1 \vee P_2 \vee P_3 \vee P_4 \vee P_5$

(2) $\text{post}^1(I_{init}, P_R) = \exists pc'', x'', y'', z'' (pc'' = l_1 \wedge pc = l_2 \wedge y'' > 0 \wedge x = x'' \wedge y = y'' \wedge z = z'')$
 $\equiv (pc = l_2 \wedge y > 0)$

$\text{post}^2(I_{init}, P_R) = \exists pc'', x'', y'', z'' (pc'' = l_2 \wedge y'' > 0 \wedge pc = l_3 \wedge x = 2y'' \wedge y = y'' \wedge z = z'')$
 $\equiv (pc = l_3 \wedge x = 2y \wedge y > 0)$

$\text{post}^3(I_{init}, P_R) = \exists pc'', x'', y'', z'' (pc'' = l_3 \wedge x'' = 2y'' \wedge y'' > 0 \wedge z = z'' \wedge x' = x'' \wedge y = y'')$
 $\equiv (pc = l_4 \wedge x = 2y \wedge z = y \wedge y > 0)$

$\text{post}^4(I_{init}, P_R) = \exists pc'', x'', y'', z'' (pc'' = l_4 \wedge x'' = 2y'' \wedge z'' = y'' \wedge y'' > 0 \wedge$
 $[x'' > z'' \wedge pc = l_5 \wedge x = x'' \wedge y = y'' \wedge z = z''] \vee$
 $[x'' \leq z'' \wedge pc = l_6 \wedge x = x'' \wedge y = y'' \wedge z = z''])$
 $\equiv (pc = l_5 \wedge x = 2y \wedge z = y \wedge y > 0 \wedge x > z) \vee$
 $(pc = l_6 \wedge x = 2y \wedge z = y \wedge y > 0 \wedge x \leq z)$

$\text{post}^5(I_{init}, P_R) = \text{post}^5(I_{init}, P_R) = \text{post}^4(I_{init}, P_R)$

(3) $P_{reach} = (pc = l_1) \vee (pc = l_2 \wedge y > 0) \vee (pc = l_3 \wedge x = 2y \wedge y > 0) \vee (pc = l_4 \wedge x = 2y \wedge z = y \wedge y > 0) \vee$
 $\vee (pc = l_5 \wedge x = 2y \wedge z = y \wedge y > 0 \wedge x > z) \vee (pc = l_6 \wedge x = 2y \wedge z = y \wedge y > 0 \wedge x \leq z)$

(4) $P_{reach} \wedge (pc = l_6) \equiv (pc = l_6 \wedge x = 2y \wedge z = y \wedge y > 0 \wedge x \leq z)$
 unsatisfiable since if $x = 2y \wedge y > 0$ then $x > y = z$