For the example on Slides 14 – 15 :

Init : $pc = l_1$ $\qquad\qquad$ φ_{error} : $pc = l_5$

$$\mathrm{Post}(\mathrm{Init}, \rho_R) = \mathrm{Post}(\mathrm{Init}, \rho_1)$$
$$= \exists pc'', x'', y'', z'' \big(pc'' = l_1 \wedge pc = l_2 \wedge y'' \geq z'' \wedge x = x'' \wedge y = y'' \wedge z = z'' \big)$$
$$\equiv (pc = l_2 \wedge y \geq z).$$

$\not\models \mathrm{Init}; \mathrm{Post}(\mathrm{Init}, \rho_R) \wedge \varphi_{error} \models \bot$

$$\mathrm{Post}^2(\mathrm{Init}, \rho_R) = \mathrm{Post}(\mathrm{Post}(\mathrm{Init}, \rho_R), \rho_R)$$
$$= \mathrm{Post}(\mathrm{Post}(\mathrm{Init}, \rho_R), \rho_2) \vee \mathrm{Post}(\mathrm{Post}(\mathrm{Init}, \rho_R), \rho_3)$$
$$= \exists pc'', x'', y'', z'' \big(pc'' = l_2 \wedge y'' \geq z'' \wedge pc = l_2 \wedge x''+1 \leq y'' \wedge x = x''+1 \wedge y = y'' \wedge z = z'' \big)$$
$$\vee$$
$$\exists pc'', x'', y'', z'' \big(pc'' = l_2 \wedge y'' \geq z'' \wedge pc = l_3 \wedge x'' \geq y'' \wedge x'' = x \wedge y = y'' \wedge z = z'' \big)$$
$$\equiv \underbrace{(pc = l_2 \wedge y \geq z \wedge x \leq y)}_{\subseteq \mathrm{Post}(\mathrm{Init}, \rho_R)} \vee (pc = l_3 \wedge x \geq y \wedge y \geq z).$$

$\not\models \mathrm{Post}^1(\mathrm{Init}, \rho_R)$

$$\underset{\mathrm{init}}{\mathrm{Post}^0} \vee \mathrm{Post}^1(\mathrm{Init}, \rho_R) \vee \mathrm{Post}^2(\mathrm{Init}, \rho_R) = (pc = l_1) \vee (pc = l_2 \wedge y \geq z) \vee \cancel{(pc = l_2 \wedge y \geq z \wedge x \leq y)} \vee (pc = l_3 \wedge x \geq y \wedge y \geq z)$$

disjoint from φ_{err} !

$\subseteq$ can be computed as before (*)

$$Post^3(init, \rho_R) = Post((pc = l_2 \wedge y \geq z), \rho_R) \vee Post((pc = l_3 \wedge x \geq y \wedge y \geq z), \rho_R) \quad **$$

(*)

$$= (pc = l_2 \wedge y \geq z \wedge x \leq y) \vee (pc = l_3 \wedge x \geq y \wedge y \geq z)$$

$$\vee$$

$$\exists pc'', x'', y'', z'' \, (pc'' = l_3 \wedge x'' \geq y'' \wedge y'' \geq z'' \wedge pc = l_4 \wedge x \geq z \wedge x = x'' \wedge y = y'' \wedge z = z'')$$

$$\vee$$

$$\exists pc'', x'', y'', z'' \, (pc'' = l_3 \wedge x'' \geq y'' \wedge y'' \geq z'' \wedge pc = l_5 \wedge x < z \wedge x = x'' \ldots)$$

$$= \underbrace{\quad}_{*} \vee (pc = l_4 \wedge x \geq y \wedge y \geq z \wedge x \geq z)$$

$$\vee (pc = l_5 \wedge x \geq y \wedge y \geq z \wedge x < z)$$

Note that the last constraint in $Post^3(init, \rho_R)$ is unsatisfiable if we assume that x, y, z are interpreted over integers and $\geq$ is the usual order relation on integers.

$$\bigvee_{i=1}^{4} Post^i(init, \rho_R) = \bigvee_{i=1}^{3} Post^i(init, \rho_R)$$

fixpoint reached.

$$\left(init \vee Post(init, \rho_R) \vee Post^2(init, \rho_R) \vee Post^3(init, \rho_R)\right)$$

$$\wedge \varphi_{err} = \Big[(pc = l_1) \vee (pc = l_2 \wedge y \geq z) \vee (pc = l_3 \wedge x \geq y \wedge y \geq z)$$

$$\vee (pc = l_4 \wedge x \geq y \wedge y \geq z \wedge x \geq z) \vee$$

$$\vee (pc = l_5 \wedge x \geq y \wedge y \geq z \wedge x < z)\Big] \wedge (pc = l_5)$$

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