

Exercises for “Formal Specification and Verification” Exercise sheet 1

Exercise 1.1:

Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

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|--|---------|
| (1) $(P \wedge \neg Q)$ | (4) Q |
| (2) $(R \vee \neg S)$ | (5) S |
| (3) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$ | (6) V |

Exercise 1.2:

Let F be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge (Q \rightarrow P) \wedge \neg P)] \wedge (P \vee R)$$

- (1) Compute the negation normal form (NNF) F' of F .
- (2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 1.3:

Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

- (1) $\neg P \vee \neg Q \vee R$
- (2) $\neg P \vee \neg Q \vee S$
- (3) P
- (4) $\neg S \vee \neg R$
- (5) Q

Exercise 1.4:

Let F be a formula, P a propositional variable not occurring in F , and F' a subformula of F .

Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Supplementary exercise (to be discussed on November 3, 2016)

Exercise 1.5:

Let F be a formula containing neither \rightarrow nor \leftrightarrow , P a propositional variable not occurring in F , and F' a subformula of F .

Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Please submit your solution until Wednesday, November 2, 2016 at 11:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework FSW” in the subject.
- Put it in the box in Room B 222.