Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **10. November 2016**

Exercises for "Formal Specification and Verification"
Exercise sheet 2

Exercise 2.1:
Let F be the following formula:

$$\neg[\ \neg(P \vee (\neg Q \vee R))\ \vee\ (\neg(P \vee Q) \vee (P \vee R))\ ]$$

Convert F to CNF using the (optimized) satisfiability-preserving transformation described in the lecture.

Exercise 2.2:
Use a DPLL procedure to find a model of each of the following formulae, or prove that the particular formula has no model:

(1) $(P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (Q \vee \neg R) \wedge (\neg Q \vee \neg R)$

(2) $(P \vee Q \vee \neg R) \wedge (P \vee \neg Q) \wedge (P \vee Q \vee R) \wedge (R \vee Q) \wedge (R \vee \neg Q) \wedge (\neg P \vee \neg R) \wedge \neg U$

Exercise 2.3:
Consider the following deductive system for propositional logic (with signature $\neg, \rightarrow$):

Axiom schemata:

(1) $\neg p \rightarrow (p \rightarrow q)$

(2) $p \rightarrow (q \rightarrow p)$

(3) $(p \rightarrow q) \rightarrow ((\neg p \rightarrow q) \rightarrow q)$

(4) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

Inference rules

Modus Ponens: $\frac{p,\quad p \rightarrow q}{q}$

Give a proof for $F \rightarrow F$ in this system.

Hint: You can e.g. use instances of axiom schema 2 (twice), 4, and Modus Ponens (twice).

Exercise 2.4:
Give a proof for

$$\Rightarrow ((P \to (Q \to R)) \to ((P \to Q) \to (P \to R)))$$

in the sequent calculus for propositional logic presented in the lecture.

Exercise 2.5:
Consider the following boolean formula $F := (P \wedge ((Q \wedge \neg R) \vee (\neg Q \wedge R)))$.

(1) Construct a reduced OBDD B_F for F with the order $[P, Q, R]$ i.e. such that the root is a P-node followed by Q- and then R-nodes.

(2) Let B_F be the OBDD constructed previously. Construct the following OBDDs:

   (a) $\mathsf{restrict}(0, R, B_F)$;

   (b) $\mathsf{restrict}(1, R, B_F)$;

   (c) $\mathsf{exists}(R, B_F)$.

You will be able to solve this exercise only after OBDDs and operations on OBDDs are introduced in the lecture on Tuesday, 15.11.2016.

Please submit your solution until Wednesday, November 16, 2016 at 11:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSW" in the subject.
- Put it in the box in Room B 222.