Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **January 12, 2017**

Exercises for "Formal Specification and Verification"
Exercise sheet 5

We use the following abbreviations in LTL:

- The future diamond $\Diamond\phi := \top\mathcal{U}\phi$
- The future box $\Box\phi := \neg\Diamond\neg\phi$
- The release operator $\phi\mathcal{R}\psi := \neg(\neg\phi\mathcal{U}\neg\psi)$

Exercise 5.1:
Let $TS = (S, \to, L)$ be a transition system and let $\pi = s_0 \to s_1 \to s_2 \to ...$ be a path in TS.

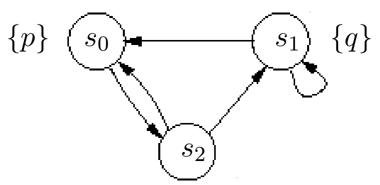
Prove:

$\pi \models \phi\mathcal{R}\psi$ if and only if $[(\forall m \geq 0 : \pi^m \models \psi)$ or $(\exists n \geq 0 : \pi^n \models \phi$ and $\forall k \leq n$: $\pi^k \models \psi)]$.

Hint: You might need to use the fact that the following are equivalent:

- $\forall m \geq 0\,(\pi^m \models \psi$ or $\exists n < m : \pi^n \models \phi)$
- $(\forall m \geq 0 : \pi^m \models \psi)$ or $(\exists n \geq 0 : \pi^n \models \phi$ and $\forall k \leq n$: $\pi^k \models \psi)$

Exercise 5.2:
Consider the following transition system:



Find an (infinite) path π in this transition system with $\pi \models p\mathcal{U}q$.

Find an (infinite) path π' in this transition system with $\pi' \models \neg(p\mathcal{U}q)$.

Exercise 5.3:
Let $TS = (S, \to, L)$ be a transition system and let $\pi = s_0 \to s_1 \to s_2 \to ...$ be a path in TS.

Show that:

(1) $\pi \models \bigcirc(\phi \to \psi) \to (\bigcirc\phi \to \bigcirc\psi)$

(2) $\pi \models \Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

(3) $\pi \models \Box\phi \to \phi \wedge \bigcirc\Box\phi$

(4) $\pi \models \Box(\phi \to \bigcirc\phi) \to (\phi \to \Box\phi)$

(5) $\pi \models \phi\mathcal{U}\psi \to (\psi \vee (\phi \wedge \bigcirc(\phi\mathcal{U}\psi)))$

Exercise 5.4:
Show that there exists no transition system $TS = (S, \to, L)$ and no path $\pi = s_0 \to s_1 \to s_2 \to ...$ in TS with $\pi \models p \wedge (p \to \Box p) \wedge \Diamond\neg p$.

Exercise 5.5:
Prove the following equivalences of LTL formulae:

(1) $\phi\mathcal{U}\phi \equiv \phi$

(2) $\bigcirc\Diamond\phi \equiv \Diamond \bigcirc \phi$

(3) $\phi\mathcal{U}\psi \equiv \psi \vee (\phi \wedge \bigcirc(\phi\mathcal{U}\psi))$ (unfolding of until)

(4) $\phi\mathcal{R}\psi \equiv (\psi \wedge \phi) \vee (\psi \wedge \bigcirc(\phi\mathcal{R}\psi))$ (unfolding of release)

Hint: For proving (4) you can use (3) and the definition of $\mathcal{R}$.

Exercise 5.6:
Consider a signature with $\Pi = \{P, Q, S\}$. Which of the following formulae are CTL formulae? Justify your answer.

(1) $\bigcirc P$

(2) $A(\bigcirc(P \wedge Q) \vee (S\mathcal{U}P))$

(3) $A(\bigcirc(P \wedge Q)) \vee E(S\mathcal{U}P)$

(4) $(A\Diamond P) \vee (\Box(EQ))$

(5) $(A\Diamond P) \vee A(\Box(EQ))$

(6) $(A\Diamond P) \vee A(E\Box Q)$

Please submit your solution until Wednesday, January 18, 2017 at 12:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework FSV” in the subject.
- Hand it in to me (Room B225) or drop it in the box in front of Room B224.