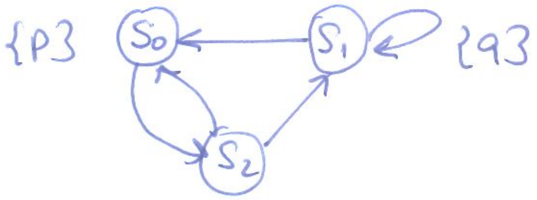


sat(E(q U p))
F G

- $W := sat(q) = \{s_1\}$
- $X := S = \{s_0, s_1, s_2\}$
- $Y := sat(p) = \{s_0\}$

$X \neq Y$:
- $X := \{s_0\}$
- $Y := Y \cup (W \cap \underline{pre_\exists(Y)}) = \{s_0, s_1\}$

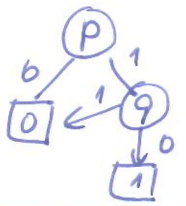
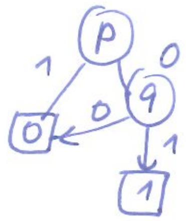
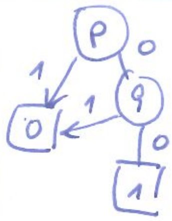
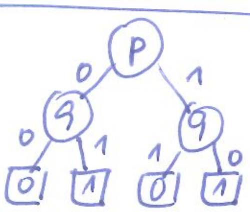
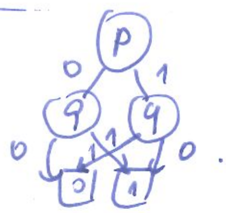
$\underline{pre_\exists(Y) = \{s_1, s_2\}}$

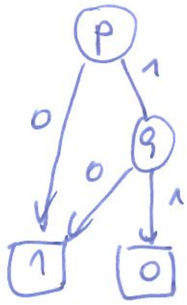
$X \neq Y$:
- $X := \{s_0, s_1\}$
- $Y := Y \cup (W \cap \underline{pre_\exists(Y)}) = \{s_0, s_1\}$.

$\underline{pre_\exists(Y) := \{s_2, s_1, s_0\}}$

$X = Y$ $sat(E(q\,U\,p)) = \{s_0, s_1\}$.

Solution using OBDDs.

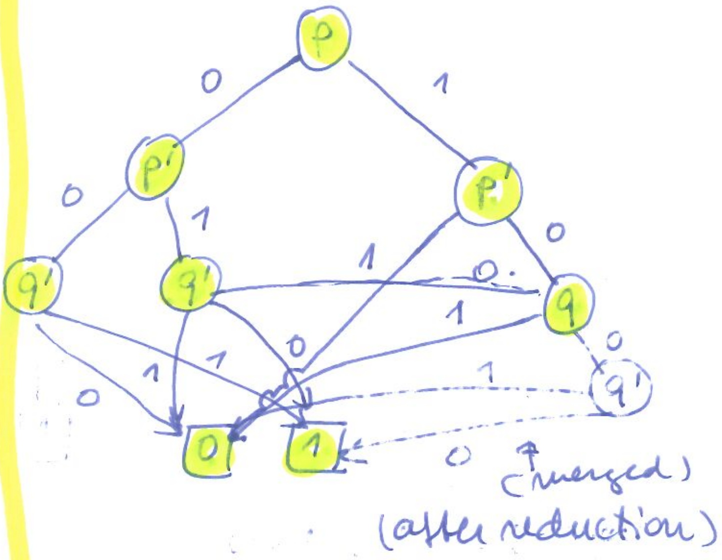
SETS	Formula	OBDD
$\emptyset$	$\perp$	$\boxed{0}$
$\{s_0\}$	$p \wedge \neg q$	
$\{s_1\}$	$\neg p \wedge q$	
$\{s_2\}$	$\neg p \wedge \neg q$	
$\{s_0, s_1\}$	$(p \wedge \neg q) \vee (\neg p \wedge q)$	 
$\{s_1, s_2\}$	$\neg p$.	

	Formula	OBDD
S	$(p \wedge \neg q) \vee$ $(\neg p \wedge q) \vee$ $(\neg p \wedge \neg q)$ $\equiv (p \wedge \neg q) \vee \neg p$ $\equiv \neg q \vee \neg p$.	

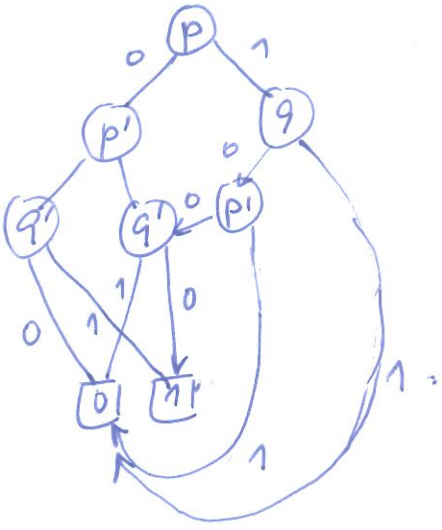
Transition relation

p	q	p'	q'	$\to$	
0	0	0	0	0	
0	0	0	1	1	$\neg p \land \neg q \land \neg p' \land q'$
0	0	1	0	1	$\neg p \land \neg q \land p' \land \neg q'$
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	1	$\neg p \land q \land \neg p' \land . q'$
0	1	1	0	1	$\neg p \land q \land p' \land \neg q'$
0	1	1	1	0	
1	0	0	0	1	$p \land \neg q \land \neg p' \land \neg q'$.
1	0	0	1	0	
1	0	1	0	.	
1	0	1	1	.	
1	1	0	0	.	
1	1	0	1	.	
1	1	1	0	.	
1	1	1	1	0.	

ordering [p,p',q,q']

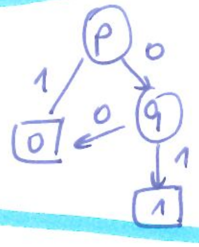


ordering [p,q,p',q']



Sat(E(q U p))

- $W := sat(q) = \{s_1\}$

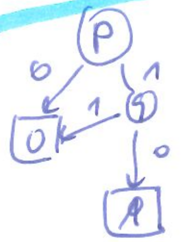


B_W

- $X := S$

B_X

- $Y := sat(q) = \{s_0\}$



B_Y.

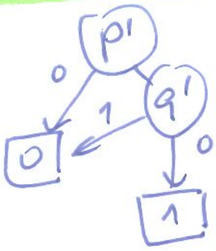
$B_X \neq B_Y$ (i.e. $X \neq Y$); $X := Y$.

- $B_Y := apply(\vee, B_Y, apply(\wedge, B_W, B_{pre_\exists(Y)}))$

→ $pre_\exists(Y) = \{s \in S \mid \exists s' (s \to s' \text{ and } s' \in Y)\}$.

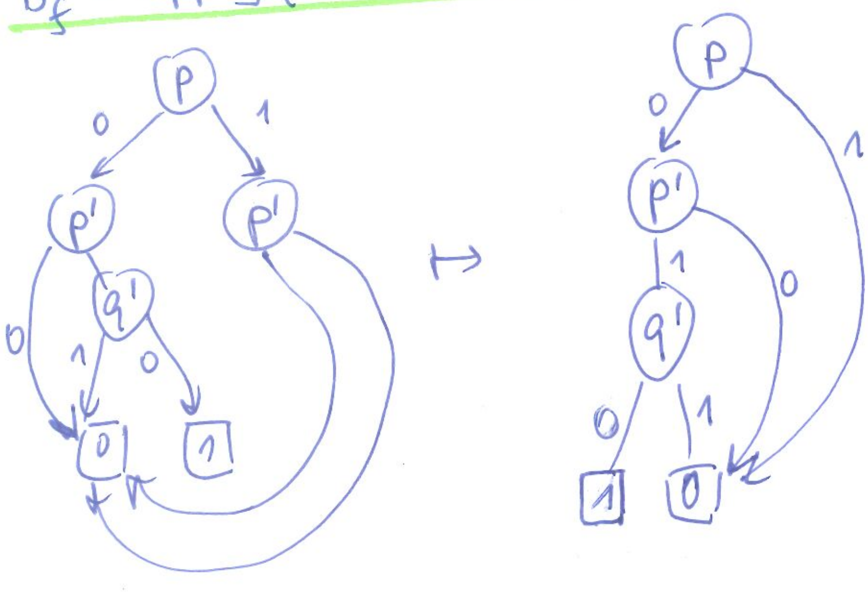
→ Rename variables in B_Y to primed versions

$B_{Y'}$:



→ Compute OBDD for $exists(p', q', apply(\wedge, B_\to, B_{Y'}))$

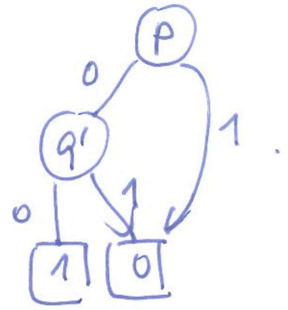
(1) $B_f = apply(\wedge, B_\to, B_{Y'})$:



(2) exists(p', q', B_f).

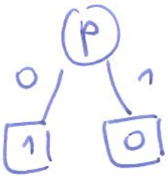
$B_{f'}$ = exists(p', B_f) = apply$(\vee,$ restrict$(0, p', B_f)$, restrict$(1, p', B_f))$

$\boxed{0}$

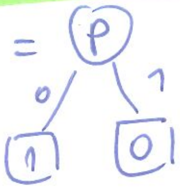


= 

exists$(q', B_{f'})$ = apply$(\vee,$ restrict$(0, q', B_{f'})$, restrict$(1, q', B_{f'}))$



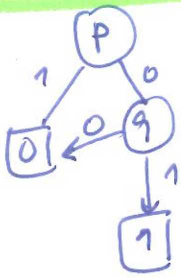
$\boxed{0}$

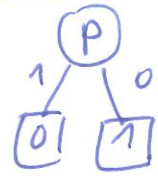
$B_{me\exists(Y)}$. = 

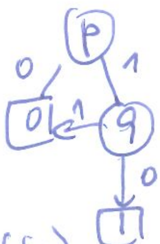
[corresponds to $\neg p$, i.e. to $\{s_1, s_2\}$.]

After having computed $B_{me\exists(Y)}$ we can compute the new B_Y:

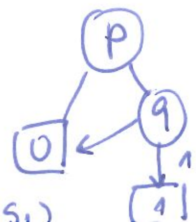
$B_Y :=$ apply$(\vee, B_Y,$ apply$(\wedge, B_W, B_{me\exists(Y)}))$.



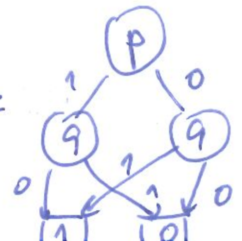


= apply$(\vee$, 

(s_0)

, 

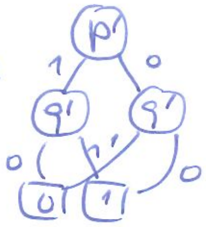
(s_1)

) = 

(OBDD for the set $\{s_0, s_1\}$)

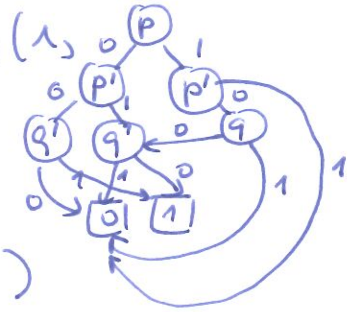
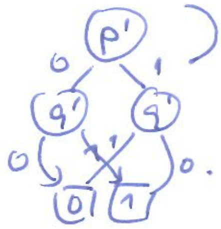
$B_X \neq B_Y$ (hence $X \neq Y$).

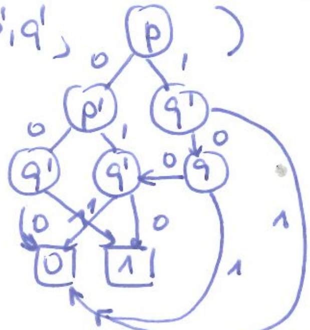
- $B_X := B_Y =$  corresponds to $\{s_0, s_1\}$.

$B_{Y'}$: 

- $B_Y := \text{apply}(\vee, B_Y, \text{apply}(\wedge, B_W, B_{pre\exists}(Y)))$.

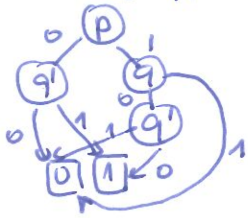
$B_{pre\exists}(Y) = \text{exists}(p', q', \text{apply}(\wedge, B_{\rightarrow}, B_{Y'}))$

$= \text{exists}(p', q', \text{apply}(\wedge,$  ,  $))$

$= \text{exists}(p', q',$  $)$ ← B'_f.

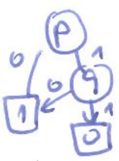
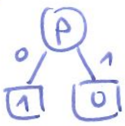
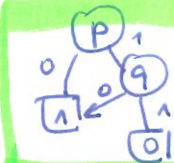
$= \text{exists}(q', \text{exists}(p', B'_f))$

$= \text{exists}(q', \text{apply}(\vee, \text{restrict}(0, p', B'_f), \text{restrict}(1, p', B'_f)))$

$= \text{exists}(q', \text{apply}(\vee,$  ,  $))$

$= \text{exists}(q',$  $)$ ← $B'_{f'}$

$= \text{apply}(\vee, \text{restrict}(0, q', B'_{f'}), \text{restrict}(1, q', B'_{f'}))$

$= \text{apply}(\vee,$  ,  $) =$  (corresp. to $\{s_0, s_1, s_2\}$) $B_{pre\exists}(Y)$

$B_Y := \text{apply}(\vee,$  , $\text{apply}(\wedge,$  ,  $))$

$=$ 

$B_X = B_Y$ Thus the OBDD for $\text{sat}(E(q\, \mathcal{U}\, p))$ is  (corresponds to $\{s_0, s_1\}$).