

①

sat($E(q \cup p)$)
F G

- $W := \text{nat}(q) = \{s_1\}$
- $X := S = \{s_0, s_1, s_2\}$
- $Y := \text{nat}(p) = \{s_0\}$

- $X \neq Y$**
- $X := \{s_0\}$
 - $Y := Y \cup (W \cap \text{me}_\exists(Y)) = \{s_0, s_1\}$

$\text{me}_\exists(Y) = \{s_1, s_2\}$

- $X \neq Y$**
- $X := \{s_0, s_1\}$
 - $Y := Y \cap (W \cap \text{me}_\exists(Y)) = \{s_0, s_1\}$

$\text{me}_\exists(Y) := \{s_2, s_1, \overline{s_0}\}$ (without s_0)

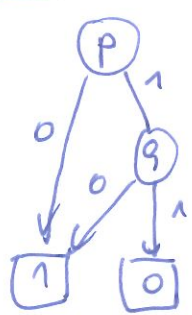
$X = Y$ $\text{nat}(E(q \cup p)) = \{s_0, s_1\}$

Solution using OBDDs.

SETS

SETS	Formula	OBDD
\emptyset	\perp	
$\{s_0\}$	$p \wedge \neg q$	
$\{s_1\}$	$\neg p \wedge q$	
$\{s_2\}$	$\neg p \wedge \neg q$	
$\{s_0, s_1\}$	$(p \wedge \neg q) \vee (\neg p \wedge q)$	
$\{s_1, s_2\}$	$\neg p$	

Formula	OBDD
S	$(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ $\equiv (p \wedge \neg q) \vee \neg p$ $\equiv \neg q \vee \neg p$



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Transition relation

(2)

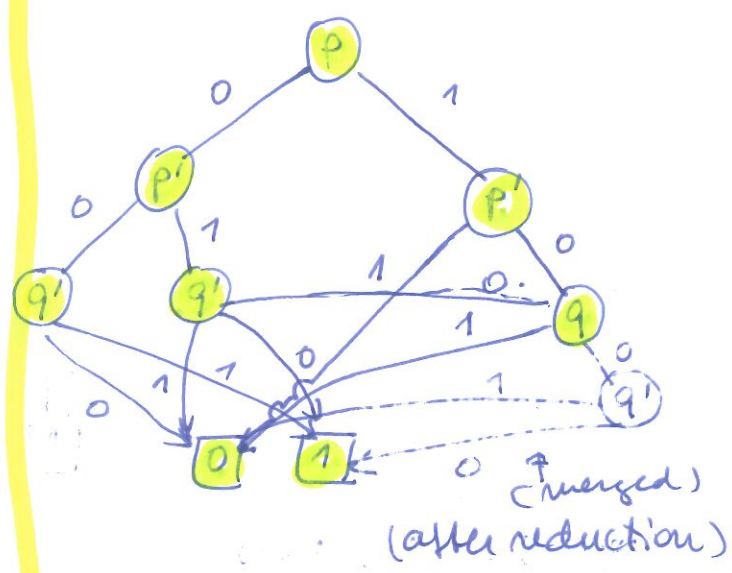
P	q	P'	q'	→
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	⋮
1	0	1	1	⋮
1	1	0	0	⋮
1	1	0	1	⋮
1	1	1	0	⋮
1	1	1	1	0.

$\neg p \wedge \neg q \wedge \neg p' \wedge q'$
 $\neg p \wedge \neg q \wedge p' \wedge \neg q'$

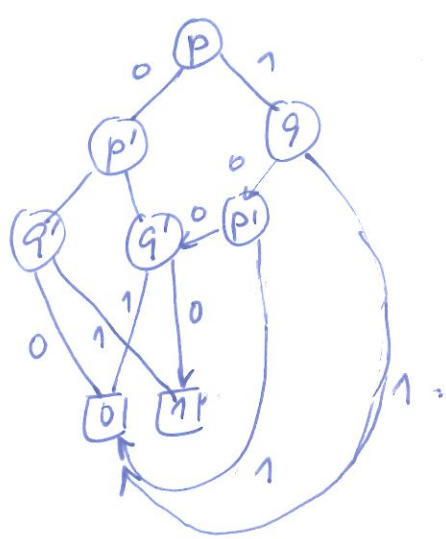
$\neg p \wedge q \wedge \neg p' \wedge q'$
 $\neg p \wedge q \wedge p' \wedge \neg q'$

$p \wedge \neg q \wedge \neg p' \wedge \neg q'$

ordering [P, P', q, q']

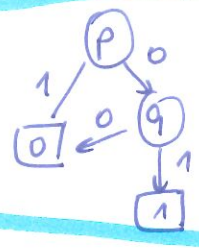


ordering [P, q, P', q']



Sat($E(q \cup p)$)

$W := \text{sat}(q) = \{S_1\}$



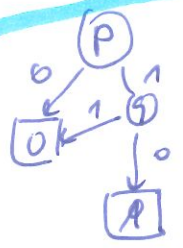
B_W

$X := S$



B_X

$Y := \text{sat}(p) = \{S_0\}$



B_Y

$B_X \neq B_Y$

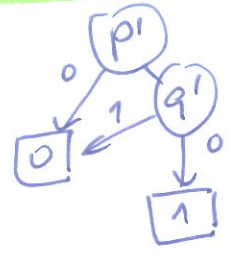
(i.e. $X \neq Y$); $X := Y$.

$B_Y := \text{apply}(v, B_Y, \text{apply}(1, B_W, B_{\text{me}} \rightarrow(Y)))$

$\text{me} \rightarrow(Y) = \{S \in S \mid \exists S' (S \rightarrow S' \text{ and } S' \in Y)\}$

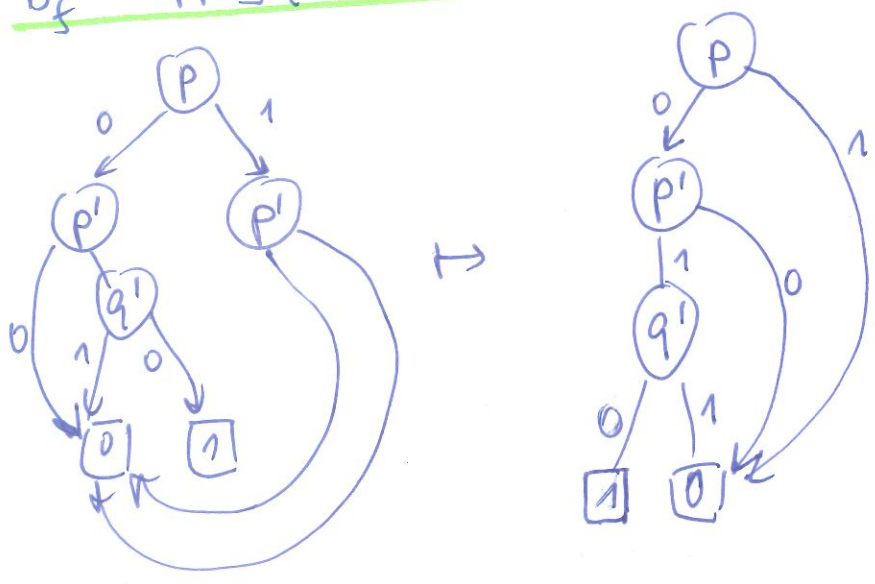
Rename variables in B_Y to primed versions

$B_{Y'}$:



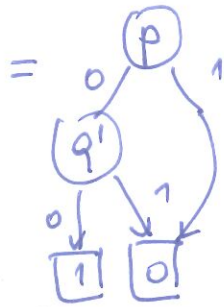
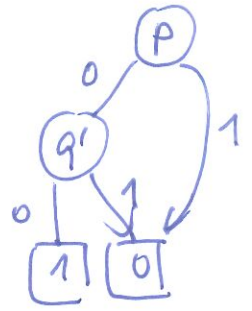
Compute OBDD for exists ($P', q', \text{apply}(1, B_{\rightarrow}, B_{Y'})$)

$B_f = \text{apply}(1, B_{\rightarrow}, B_{Y'})$:

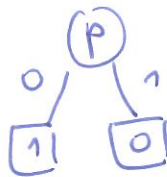


(2) exists(p', q', B_f).

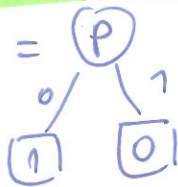
B_f' = exists(p', B_f) = apply(v, restrict(0, p', B_f), restrict(1, p', B_f))



exists(q', B_f') = apply(v, restrict(0, q', B_f'), restrict(1, q', B_f'))



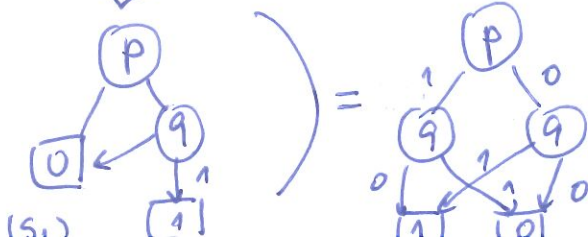
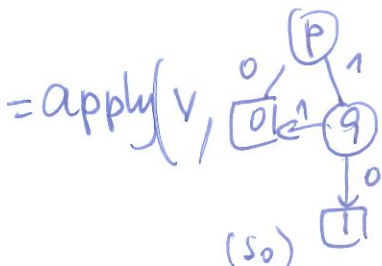
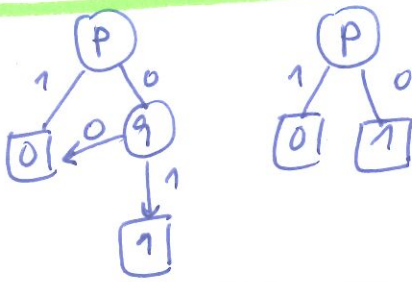
B_{me \exists(x)}



corresponds to γ_p ,
i.e. to $\{s_1, s_2\}$.

After having computed B_{me \exists(x)} we can compute the new B_f:

B_f = apply(v, B_f, apply(1, B_w, B_{me \exists(x)}))



(OBDD for the set $\{s_0, s_1\}$)

Since B_X and B_Y are different (hence X and Y are different) one more iteration is necessary:

$X := Y \quad (B_X := B_Y)$

$Y := Y \text{ union } (W \text{ intersection } \text{pre_exists}(Y))$

Complete the last steps of the algorithm to compute the OBDD describing $\text{sat}(E(qUp))$.