Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **13. November 2016**

Exercises for "Formal Specification and Verification"
Exercise sheet 4

Exercise 4.1:
Consider the boolean formula discussed in Exercise 3, $F := (P \wedge ((Q \wedge \neg R) \vee (\neg Q \wedge R)))$. Let B_F be the OBDD for F constructed previously. Construct the following OBDDs:

(a) $\mathsf{restrict}(0, R, B_F)$;

(b) $\mathsf{restrict}(1, R, B_F)$;

(c) $\mathsf{exists}(R, B_F)$.

Exercise 4.2:
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae, which are neither?

(a) $\neg p(g(a), f(x, y))$

(b) $f(x, x) \approx x$

(c) $p(f(x, a), x) \vee p(a, b)$

(d) $p(\neg g(x), g(y))$

(e) $\neg p(f(x, y))$

(f) $p(a, b) \wedge p(x, y) \wedge y$

(g) $\exists y(\neg p(f(y, y), y))$

(h) $\forall x \forall y(g(p(x, y)) \approx g(x))$

Exercise 4.3:
Let $\Sigma = (S, \Omega, \Pi)$ be a many-sorted signature, where $S = \{\mathsf{int}, \mathsf{list}\}$, $\Omega = \{\mathsf{cons}, \mathsf{car}, \mathsf{cdr}, \mathsf{nil}, b\}$ and $\Pi = \{p\}$ with the following arities:

$a(\mathsf{cons}) = \mathsf{int}, \mathsf{list} \to \mathsf{list}$ $\quad a(\mathsf{car}) = \mathsf{list} \to \mathsf{int}$ $\quad a(\mathsf{cdr}) = \mathsf{list} \to \mathsf{list}$
$a(\mathsf{nil}) = \to \mathsf{list}$ (i.e. nil is a constant of sort list)
$a(b) = \to \mathsf{int}$ (i.e. b is a constant of sort int)
$a(p) = \mathsf{int}, \mathsf{list}$.

Let X_{int} be the set of variables of sort int containing $\{i, j, k\}$, and let X_{list} be the set of variables of sort list containing $\{x, y, z\}$. Let $X = \{X_{\mathsf{int}}, X_{\mathsf{list}}\}$.

Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae (in first-order logic with equality, where equality between terms of sort int is $\approx_i$ and equality between terms of sort list is $\approx_l$), which are neither?

(a) $\mathsf{cons}(\mathsf{cons}(b, \mathsf{nil}), \mathsf{nil})$
(b) $\mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil}))$
(c) $\neg p(b, \mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})))$
(d) $\neg p(\mathsf{cons}(b, \mathsf{nil}), \mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})))$
(e) $\mathsf{cons}(b, \mathsf{cons}(b, \mathsf{nil})) \approx_l \mathsf{cons}(\mathsf{cons}(x, b), \mathsf{nil})$
(f) $\mathsf{cons}(i, \mathsf{cons}(b, \mathsf{nil})) \approx j$
(g) $p(\neg\mathsf{car}(x), x)$
(h) $\neg p(\mathsf{car}(x), x) \vee p(j, \mathsf{cons}(j, x))$
(i) $\neg p(b, x) \vee p(b, \mathsf{cons}(b, x)) \vee b$
(j) $\forall i : \mathsf{int}, \forall x : \mathsf{list}\ (\mathsf{cons}(\mathsf{car}(x), \mathsf{cdr}(x)) \approx_l x)$
(k) $\exists i : \mathsf{int}, \forall y : \mathsf{list}\ (\mathsf{cons}(b, p(x, y)) \approx_l \mathsf{cdr}(y))$

Exercise 4.4:
Compute the results of the following substitutions:

(a) $f(g(x), x)[g(a)/x]$
(b) $p(f(y, x), g(x))[x/y]$
(c) $\forall y(p(f(y, x), g(y)))[x/y]$
(d) $\forall y(p(f(y, x), x))[y/x]$
(e) $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$
(f) $\exists y\big(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x)\big)[g(y)/y, g(z)/x]$

Exercise 4.5:
Let $\Sigma = (\Omega, \Pi)$, where $\Omega = \{0/0, s/1, +/2\}$ and $\Pi = \emptyset$ (i.e. the only predicate symbol is $\approx$). Consider the following formulae in the signature Σ:

1. $F_1 = \forall x\ (x + 0 \approx x)$
2. $F_2 = \forall x, y\ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y\ \ (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

Please submit your solution until Sunday, November 18, 2018 at 17:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework FSV” in the subject.
- Put it in the box in Room B 222.