Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Formal Specification and Verification"
Exercise sheet 5

Exercise 5.1:
$\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0, f/1\}$ and $\Pi = \{p/1\}$.

(1) How many different Herbrand interpretations over Σ exist? Explain briefly.

(2) Consider the formula $F := p(f(f(b))) \wedge \forall x\, (p(x) \rightarrow p(f(x)))$. How many different Herbrand models over Σ does the formula F have? Explain briefly.

(3) Every Herbrand interpretation which is a model of F is also a model of $G := \forall x\, p(f(f(x)))$. Give an example of an algebra that is a model of F but not of G.

(4) Let $\mathcal{A}$ be a Herbrand interpretation over Σ and let $\sim$ be the binary relation on T_Σ defined by:

$$t_1 \sim t_2 \text{ iff } \forall x\, (f(f(f(x))) \approx x) \models t_1 \approx t_2.$$

- Is $\sim$ a congruence relation on $\mathcal{A}$?
- Describe the quotient structure $\mathcal{A}/\!\sim$.
- Describe the class $\{\mathcal{A}/\!\sim \mid \mathcal{A} \text{ Herbrand interpretation over } \Sigma\}$.

Exercise 5.2:
Consider the following specification of binary trees (in a variant of the CASL syntax)

spec BinTree =

sort	elem, tree
operations	$a :\rightarrow$ elem
	empty $:\rightarrow$ tree
	leaf : elem $\rightarrow$ tree
	make : tree, tree $\rightarrow$ tree
	right : tree $\rightarrow$ tree
	left : tree $\rightarrow$ tree
Axioms:	$\forall x_1, x_2$: tree, $\forall e$: elem:
	• right(empty) $\approx$ empty
	• right(leaf(e)) $\approx$ empty
	• left(empty) $\approx$ empty
	• left(leaf(e)) $\approx$ empty
	• left(make(x_1, x_2)) $\approx x_1$
	• right(make(x_1, x_2)) $\approx x_2$

(1) Let $\mathcal{F}$ be the set of axioms in the specification above. Which of the following hold?

(1a) $\mathcal{F} \models \mathsf{left}(\mathsf{make}(\mathsf{empty}, \mathsf{empty})) \approx \mathsf{empty}$

(1b) $\mathcal{F} \models \mathsf{make}(x_1, x_2) \approx \mathsf{empty}$

(1c) $\mathcal{F} \models (x_2 \approx \mathsf{empty} \wedge x_3 \approx \mathsf{make}(x_1, \mathsf{empty})) \rightarrow \mathsf{make}(\mathsf{left}(\mathsf{make}(x_1, x_2)), \mathsf{right}(\mathsf{leaf}(e)) \approx x_3$

(1d) $\mathcal{F} \models \mathsf{make}(x_1, \mathsf{make}(x_2, x_3)) \approx x_2$

(2) Let $\sim$ be defined on T_Σ by:

$$t_1 \sim t_2 \text{ iff } \mathcal{F} \models t_1 \approx t_2.$$

Describe the quotient algebra $\mathcal{T}_\Sigma/\sim$.

(3) Let $\sim'$ be defined on T_Σ by

$$t_1 \sim' t_2 \text{ iff } (\mathcal{F} \cup \{\forall x \, \mathsf{left}(x) \approx \mathsf{right}(x)\} \models t_1 \approx t_2).$$

Describe the quotient algebra $\mathcal{T}_\Sigma/\sim'$.

Please submit your solution until Sunday, November 25, 2018 at 17:00. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework FSV" in the subject.
- Drop it in the box in Room B222.