Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 10

Exercise 10.1: (*6 P*)
Compute the translation into first order logic used for checking the validity of a modal formula Φ (of the form $\exists x P_{\neg\Phi}(x) \wedge \mathsf{Rename}(\neg\Phi)$) for the following formulae:

(1) $\Phi_1 : \quad (\Diamond P \vee \Diamond Q) \rightarrow \Diamond(P \vee Q)$

(2) $\Phi_2 : \quad \Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$

Exercise 10.2: (*3 P*)
Compute the translation into first order logic (of the form $\exists x P_{\Phi}(x) \wedge \mathsf{Rename}(\Phi)$) which can be used for checking the satisfiability of the formula:

$$\Phi : \quad ((\Box\Diamond P \wedge \Diamond P) \rightarrow \Diamond\Box P)$$

Exercise 10.3: (*1 P*)
Let F be a formula in propositional modal logic, P a propositional variable not occurring in F, and F' a subformula of F. Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable in a Kripke structure $\mathcal{K} = (S, R, I)$ if and only if $F[P]$ is satisfiable in the Kripke structure $\mathcal{K} = (S, R, I')$, with $I' : (\Pi \cup \{P\}) \times S \rightarrow \{0, 1\}$ is s.t. $I'(Q, s) = I(Q, s)$ for every $Q \in \Pi$ and every $s \in S$, and $I'(P, s) \leq val_{\mathcal{K}}(F')(s)$ (i.e. for every $s \in S$, $(val_{\mathcal{K}}(P)(s) \rightarrow_{\mathsf{Bool}} val_{\mathcal{K}}(F')) = 1$).
- If F' has negative polarity in F then $F[F']$ is satisfiable in a Kripke structure $\mathcal{K} = (S, R, I)$ if and only if $F[P]$ is satisfiable in the Kripke structure $\mathcal{K} = (S, R, I')$, with $I' : (\Pi \cup \{P\}) \times S \rightarrow \{0, 1\}$ is s.t. $I'(Q, s) = I(Q, s)$ for every $Q \in \Pi$ and every $s \in S$, and $I'(P, s) \geq val_{\mathcal{K}}(F')(s)$ (i.e. for every $s \in S$, $(val_{\mathcal{K}}(F')(s) \rightarrow_{\mathsf{Bool}} val_{\mathcal{K}}(P)) = 1$).

Please submit your solution until Wednesday, January 25, 2012. Please do not forget to write your name on your solution.