

Exercises for “Non-Classical Logics” Exercise sheet 2

Exercise 2.1: (4 P)

Use a tableau procedure to prove the satisfiability or unsatisfiability of the following formulae:

- (1) $(Q \rightarrow P) \wedge (P \rightarrow Q) \wedge (R \rightarrow Q) \wedge (Q \rightarrow \neg R)$
- (2) $(R \rightarrow (P \vee Q)) \wedge (Q \rightarrow (P \wedge R)) \wedge (R \vee Q) \wedge (P \rightarrow \neg R)$

Exercise 2.2: (1 P)

Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2, g/1, a/0, b/0\}$ and $\Pi = \{p/2\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X , which are atoms/literals/clauses/formulae, which are neither?

- (a) $\neg p(g(a), f(x, y))$
- (b) $f(x, x) \approx x$
- (c) $p(f(x, a), x) \vee p(a, b)$
- (d) $p(\neg g(x), g(y))$
- (e) $\neg p(f(x, y))$
- (f) $p(a, b) \wedge p(x, y) \wedge y$
- (g) $\exists y(\neg p(f(y, y), y))$
- (h) $\forall x \forall y (g(p(x, y)) \approx g(x))$

Exercise 2.3: (2 P)

Compute the results of the following substitutions:

- (a) $f(g(x), x)[g(a)/x]$
- (b) $p(f(y, x), g(x))[x/y]$
- (c) $\forall y(p(f(y, x), g(y)))[x/y]$
- (d) $\forall y(p(f(y, x), x))[y/x]$
- (e) $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(g(z) \approx y))[g(b)/z]$
- (f) $\exists y(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x))[g(y)/y, g(z)/x]$

Exercise 2.4: (3 P)

Reminder: A formula F is valid in a Σ -algebra \mathcal{A} under assignment β (Notation: $\mathcal{A}, \beta \models F$) if $\mathcal{A}(\beta)(F) = 1$. F is valid in \mathcal{A} (Notation: $\mathcal{A} \models F$) iff $\mathcal{A}, \beta \models F$, for all $\beta \in X \rightarrow U_{\mathcal{A}}$.

Let $\Sigma = \{\Omega, \Pi\}$ where $\Omega = \{0, s, +\}$ and $\Pi = \{\approx\}$.

Consider the following formulae in the signature Σ :

1. $F_1 = \forall x (x + 0 \approx x)$
2. $F_2 = \forall x, y (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y (x + y \approx y + x)$.

Find a Σ -structure in which F_1 and F_2 are valid but F_3 is not.

Please submit your solution until Wednesday, November 9, 2011 in the evening. Please do not forget to write your name on your solution.

Submission possibility:

- Hand the solution in at the lecture.
- By e-mail to sofronie@uni-koblenz.de with the keyword “Homework Non-Classical Logics” in the subject.