Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

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Exercises for "Non-Classical Logics"
Exercise sheet 4

Exercise 4.1: (*2 P*)
Prove or refute the following statements (in classical logic):

(a) If F and G are first-order formulae and x is a variable then $\forall x(F \wedge G) \models \forall xF \wedge \forall xG$ and $\forall xF \wedge \forall xG \models \forall x(F \wedge G)$.

(b) If F and G are first-order formulae and x is a variable then $\exists x(F \wedge G) \models \exists xF \wedge \exists xG$ and $\exists xF \wedge \exists xG \models \exists x(F \wedge G)$.

Exercises 4.2–4.4 refer to the many-valued logic $\mathcal{L}_3$. Let $\Sigma = (\Omega, \Pi)$ be a first-order signature, let $M = \{0, u, 1\}$ and $\mathcal{F}$ be the set of connectives of $\mathcal{L}_3$, and $\mathcal{Q} = \{\forall, \exists\}$.
We will consider first-order formulae over this signature.

Exercise 4.2: (*2 P*)
Prove that the following formulae are $\mathcal{L}_3$ tautologies:

(a) $\neg(\exists xF) \ \mathsf{id} \ \forall x(\neg F)$.

(b) $\sim (\forall xF) \ \mathsf{id} \ \exists x(\sim F)$.

(c) $\neg(\exists xF) \ \mathsf{id} \ \forall x(\neg F)$.

where the truth table of id is presented in the slides of lecture 6 (17.11.2011) on page 15.

Exercise 4.3: (*2 P*)
Prove:

(a) For every term t, $\forall xq(x) \subset q(x)[t/x]$ is an $\mathcal{L}_3$-tautology.

(b) $\forall xq(x) \rightarrow q(x)[t/x]$ is not an $\mathcal{L}_3$-tautology.

Exercise 4.4: (*4 P*)
We say that an $\mathcal{L}_3$-formula F is a non-tautology iff for every 3-valued structure, $\mathcal{A}$ and every valuation $\beta : X \rightarrow \mathcal{A}$, we have $\mathcal{A}(\beta)(F) \neq 1$.

We say that an $\mathcal{L}_3$-formula F is two-valued iff for every 3-valued structure, $\mathcal{A}$ and every valuation $\beta, \mathcal{A}(\beta)(F) \in \{0, 1\}$.

Which of the following statements are true? Justify your answer.

(a) If $F \equiv G$ is a tautology and F is a tautology then G is a tautology.

(b) If $F \equiv G$ is a tautology and F is satisfiable then G is satisfiable.

(c) If $F \equiv G$ is a tautology and F is a non-tautology then G is a non-tautology.

(d) If $F \equiv G$ is a tautology and F is two-valued then G is two-valued.

Please submit your solution until Tuesday, November 24, 2011 at the lecture. Please do not forget to write your name on your solution.

Submission possibility:

- Hand the solution in at the lecture.
- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.