Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 5

The following exercises refer to the many-valued logic $\mathcal{L}_3$. Let $\Sigma = (\Omega, \Pi)$ be a first-order signature, let $M = \{0, u, 1\}$ and $\mathcal{F}$ be the set of connectives of $\mathcal{L}_3$, and $\mathcal{Q} = \{\forall, \exists\}$.

Exercise 5.1: *(3 P)*
Use the semantic tableau calculus for the many-valued logic $\mathcal{L}_3$ to prove that the following formula is an $\mathcal{L}_3$ tautology:

$$\neg(\exists x(p(x) \wedge q(x))) \ \mathsf{id} \ \forall x(\neg(p(x) \wedge q(x))).$$

(*Hint:* To avoid the problem of having to use the definition of the operator id the truth table of id can be used for devising suitable expansion rules for the tableau calculus.)

Exercise 5.2: *(4 P)*
Use the optimized translation to clause form described in the course for computing the CNF for the following signed formulae:

(a) $\{0\}{:}(P \supset Q)$

(b) $\{u\}{:}(P \supset Q)$

(c) $\{1\}{:}(P \supset Q)$

(d) $\{0, 1\}{:}(\sim (P \Rightarrow Q))$

Exercise 5.3: *(3 P)*
Check the unsatisfiability of the set $\{C_1, C_2, C_3\}$ of signed clauses by using the signed propositional resolution calculus.

$C_1 = \{0\}{:}P$
$C_2 = \{0, u\}{:}Q \vee \{1\}{:}P$
$C_3 = \{1\}{:}Q \vee \{u\}{:}R$
$C_4 = \{0, 1\}{:}R$

Please submit your solution until Wednesday, November 30, 2011. Please do not forget to write your name on your solution.