Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 7

Exercise 7.1: *(2 P)*
Consider the inference system for the logic K described in the lecture:

Axioms:

All axioms of propositional logic

$(K) \quad \Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$

Inference rules

$$\frac{A \quad A \rightarrow B}{B} \qquad \text{[Modus ponens]}$$

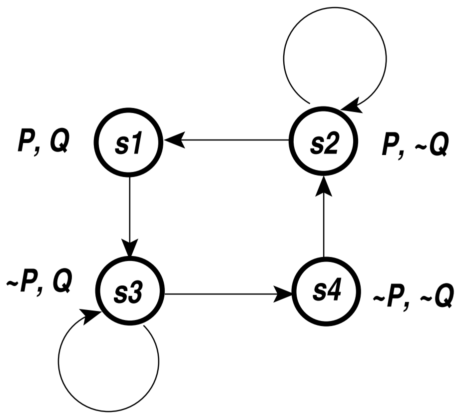
$$\frac{A}{\Box A} \qquad \text{[G]}$$

Prove that $\{Q \rightarrow P, \Box Q\} \vdash \Box(P \wedge Q)$ by constructing a proof of $\Box(P \wedge Q)$ from $N = \{Q \rightarrow P, \Box Q\}$.

Hint: In the proof you can use the following facts:

- in propositional logic $Q \rightarrow P$ is equivalent to $Q \rightarrow (P \wedge Q)$;
- use this equivalence and rule [G] to prove $\Box(Q \rightarrow (P \wedge Q))$ from N;
- use the axiom schema (K), with $A := Q$ and $B := (P \wedge Q)$;
- apply two times the [Modus ponens] rule.

Exercise 7.2: (*6 P*)
Consider the Kripke model $\mathcal{K} = (S, R, I)$ described below:



(1) Compute:

- $val_{\mathcal{K}}(\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_2)$

(2) Show that at every state $s \in S$ the following hold:

- $val_{\mathcal{K}}(P \wedge Q) \rightarrow \Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((\neg P \wedge \neg Q) \rightarrow \Box P)(s) = 1$
- $val_{\mathcal{K}}((\neg P \wedge \neg Q) \rightarrow \Box\Box P)(s) = 1$

(3) Find a state $s \in S$ such that

$$val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\Box\neg P)(s) = 0.$$

Exercise 7.3: (*2 P*)

(1) Let $\mathcal{K} = (S, R, V)$, and let $s \in S$. Show that if $\{s' \in S \mid sRs'\} = \emptyset$ (i.e. if there is no s' with sRs') then $val_{\mathcal{K}}(\Box F)(s) = 1$ for any formula F.

(2) Find a Kripke frame $\mathcal{F} = (S, R)$ with the property that for every Kripke model $\mathcal{K} = (S, R, I)$, $val_{\mathcal{K}}(\Box \perp)(s) = 1$ for every $s \in S$.

Please submit your solution until Wednesday, December 14, 2011. Please do not forget to write your name on your solution.