Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

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Exercises for "Non-Classical Logics"
Exercise sheet 8

Exercise 8.1: *(4 P)*
Prove that the following formulae are valid (i.e. true in all frames):

(1) $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$

(2) $\Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$

Exercise 8.2: *(2 P)*
Prove that in any Kripke structure $\mathcal{K}$ the following hold:

(1) If A is a propositional tautology then $\mathcal{K} \models A$.

(2) If $\mathcal{K} \models A$ and $\mathcal{K} \models A \rightarrow B$, then $\mathcal{K} \models B$.

(3) If $\mathcal{K} \models A$ then $\mathcal{K} \models \Box A$.

Exercise 8.3: *(2 P)*
Give a property of R that is necessary and sufficient for $\mathcal{F} = (S, R)$ to validate the schema $A \rightarrow \Box A$.

Do the same for $\Box \bot$.

Exercise 8.4: *(2 P)*
Let $\mathcal{K} = (S, R, I)$ be a Kripke structure and let $\mathcal{K}^* = (S^*, R^*, I^*)$ be the irreflexive Kripke structure constructed in the lecture as follows:

- For every $s \in S$ let $s^1, s^2 \notin S$ (different).
- $S^* = \{s^i \mid s \in S, i = 1, 2\}$
- $I^*(s^i, P) = I(s, P)$ for $i = 1, 2$
- $R^*(s^i, u^j)$ iff $R(s, u)$, for all i, j if $s \neq u$
- $R^*(s^i, s^j)$ iff $R(s, s)$ and $i \neq j$.

Prove by simultaneous structural induction that for every formula F and every $s \in S$ the following are equivalent:

(1) $(\mathcal{K}, s) \models F$
(2) $(\mathcal{K}^*, s^1) \models F$
(3) $(\mathcal{K}^*, s^2) \models F$

(*Remark:* Part of the proof was discussed during the lecture on December 21, 2011. It is sufficient if you complete the proof given during the lecture).

Supplementary exercise:

We will formalize the "wise men" example given in the beginning of the chapter on modal logic and show how to derive information about the knowledge of the wise men in the corresponding inference system.

Please submit your solution until Wednesday, January 11, 2012. Please do not forget to write your name on your solution.