# Universität Koblenz-Landau

**FB 4 Informatik**

---

**Prof. Dr. Viorica Sofronie-Stokkermans** **October 18, 2012**

**Exercises for "Non-Classical Logics"**
**Exercise sheet 1**

**Exercise 1.1:** *(2 P)*
Determine which of the following formulas are valid/satisfiable/unsatisfiable:

(1) $(P \wedge Q) \rightarrow P$

(2) $Q \wedge \neg Q$

(3) $\neg(\neg P \vee \neg\neg P)$

(4) $\big((Q \rightarrow P) \wedge (R \rightarrow \neg P)\big) \rightarrow (\neg Q \vee \neg R)$

**Exercise 1.2:** *(2 P)*
Prove Prop. 1.3 (2): If $N$ is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

**Definition:** A set of propositional formulas is unsatisfiable, if and only if for every valuation $\mathcal{A}$ there is a formula $G$ in the set such that $\mathcal{A} \not\models G$ (i.e. if and only if there is no valuation $\mathcal{A}$ such that $\mathcal{A} \models G$ for all formulae $G$ in the set).

**Exercise 1.3:** *(3 P)*
Let $F$ be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge \neg P)] \wedge (P \vee R)$$

(1) Compute the negation normal form (NNF) $F'$ of $F$.

(2) Convert $F'$ to CNF using:

    (a) distributivity of disjunctions over conjunctions?

    (b) the satisfiability-preserving transformation described in the lecture.

**Exercise 1.4:** (*2 P*)

Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

$$
\begin{array}{cl}
(1) & \neg P \vee \neg Q \vee R \\
(2) & \neg P \vee \neg Q \vee S \\
(3) & P \\
(4) & \neg S \vee \neg R \\
(5) & Q
\end{array}
$$

**Exercise 1.5:** (*2 P*)

Assume $S \succ P \succ Q \succ R$. Let $N$ be the following set of clauses:

$$
\begin{array}{cl}
(1) & \neg Q \vee \neg P \\
(2) & R \vee P \\
(3) & Q \vee S \\
(4) & \neg Q \vee \neg S
\end{array}
$$

- Which literals are maximal in the clauses of $N$?

- Let $S$ be the selection function which selects the negative literal $\neg Q$ in the clauses (1) and (4).

  Which inferences are possible in the ordered resolution calculus with selection $\mathsf{Res}_S^{\succ}$?

**Supplementary exercise:**
(will be discussed in the exercise session)

**Exercise 1.6:** (*2 P*)

Consider the formulae $F_n = \bigvee_{i=1}^{n} (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.

As a function of $n$, how many clauses are in:

(1) the CNF formula $F'$ constructed using the distributivity of disjunctions over conjunctions?

(2) the CNF formula $F''$ obtained using the satisfiability-preserving translation to clause form?

(3) For which $n$ is the first approach better?