Universität Koblenz-Landau FB 4 Informatik

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Exercises for "Non-Classical Logics" Exercise sheet 1

Exercise 1.1: (2 P) Determine which of the following formulas are valid/satisfiable/unsatisfiable:

- (1) $(P \land Q) \to P$
- (2) $Q \wedge \neg Q$
- (3) $\neg(\neg P \lor \neg \neg P)$
- $(4) ((Q \to P) \land (R \to \neg P)) \to (\neg Q \lor \neg R)$

Exercise 1.2: (2 P)

Prove Prop. 1.3 (2): If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

Definition: A set of propositional formulas is unsatisfiable, if and only if for every valuation \mathcal{A} there is a formula G in the set such that $\mathcal{A} \not\models G$ (i.e. if and only if there is no valuation \mathcal{A} such that $\mathcal{A} \models G$ for all formulae G in the set).

Exercise 1.3: (3 P)Let F be the following formula:

 $\neg[((Q \land \neg P) \land \neg(Q \land R)) \to (Q \land \neg P)] \land (P \lor R)$

- (1) Compute the negation normal form (NNF) F' of F.
- (2) Convert F' to CNF using:
 - (a) distributivity of disjunctions over conjunctions?
 - (b) the satisfiability-preserving transformation described in the lecture.

Exercise 1.4: (2 P)

Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

$$\begin{array}{ll} (1) & \neg P \lor \neg Q \lor R \\ (2) & \neg P \lor \neg Q \lor S \\ (3) & P \\ (4) & \neg S \lor \neg R \\ (5) & Q \end{array}$$

Exercise 1.5: (2 P)

Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

(1)	$\neg Q \lor \neg P$
(2)	$R \vee P$
(3)	$Q \vee S$
(4)	$\neg Q \vee \neg S$

- Which literals are maximal in the clauses of N?
- Let S be the selection function which selects the negative literal $\neg Q$ in the clauses (1) and (4).

Which inferences are possible in the ordered resolution calculus with selection Res_S^{\succ} ?

Supplementary exercise:

(will be discussed in the exercise session)

Exercise 1.6: (2 P)

Consider the formulae $F_n = \bigvee_{i=1}^n (Q_i \wedge R_i)$ for $n \in \mathbb{N}$. As a function of n, how many clauses are in:

- (1) the CNF formula F' constructed using the distributivity of disjunctions over conjunctions?
- (2) the CNF formula F'' obtained using the satisfiability-preserving translation to clause form?
- (3) For which n is the first approach better?

Please submit your solution until Monday, October 22, 2012, at 17:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222 (if you prefer to submit the written exercise like this please tell me such that I can prepare such a box).