

### Exercises for “Non-Classical Logics” Exercise sheet 11

#### Exercise 11.1: (4 P)

Snomed CT is a comprehensive clinical healthcare terminology that comprises more than 400 000 vocabulary items and almost the same number of logical axioms. An example of concept which is defined in Snomed CT is *apendicitis*.

Assume that we have:

- the concept names: *Disease*, *Appendicitis*, *Inflammation*, *Appendix* and
- the role names: *AssociatedMorphology*, *FindingSite*.

- (1) An example of a typical Snomed CT axiom mentions that *apendicitis* is a disease which is morphologically an (= some) inflammation. In ALC this can be expressed as follows:

$$\text{Appendicitis} \sqsubseteq \text{Disease} \sqcap \exists \text{AssociatedMorphology}.\text{Inflammation}$$

How can this be expressed in first-order logic?

- (2) How can one express in ALC the fact that *apendicitis* is a disease which is located in the (= some) appendix?

#### Exercise 11.2: (6 P)

We suppose that in the set of all possible objects there is a set of objects that are trees and a binary relation *direct-subtree* between objects that leads from a tree to its direct subtrees.

Then the binary trees are the trees with at most two direct subtrees and such that all these direct subtrees are themselves binary trees.

- (1) Write a formal description of the concept of *binary tree* in SHIQ. Indicate the set of concepts and roles which you used for this.
- (2) Assume that we additionally consider the *proper-subtree* relation, with the following properties:
  - Every direct subtree of a tree  $T$  is a proper subtree of  $T$ .
  - If  $T_1$  is a proper subtree of  $T_2$  and  $T_2$  is a proper subtree of  $T_3$  then  $T_1$  is a proper subtree of  $T_3$  (transitivity).

Present a SHIQ formalism in which you can also consider this relation, by specifying:

- a set  $N_C$  of concept names;
- a set  $N_R^0$  of atomic role symbols which you need for the specification;
- a subset  $N_t^0 \subseteq N_R^0$  of transitive role symbols needed for the specification;
- a hierarchy on roles.

Is the description of the concept of a binary tree you gave in (1) a correct SHIQ concept description over this extended language? Justify your answer.

**Exercise 11.3:** (4 P)

Show that the following formulae are valid in propositional dynamic logic (i.e. true in all PDL Kripke models) :

- (1)  $[\alpha](A \wedge B) \leftrightarrow [\alpha]A \wedge [\alpha]B$
- (2)  $[\alpha; \beta]A \leftrightarrow [\alpha][\beta]A$
- (3)  $[\alpha \cup \beta]A \leftrightarrow [\alpha]A \wedge [\beta]A$
- (4)  $[A?]B \leftrightarrow (A \rightarrow B)$
- (5)  $[\alpha^*]A \leftrightarrow A \wedge [\alpha][\alpha^*]A$
- (6)  $[\alpha^*](A \rightarrow [\alpha]A) \rightarrow (A \rightarrow [\alpha^*]A)$

Please submit your solution until Tuesday, January 29, 2013, 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with “Homework Non-Classical Logics” in subject.
- Put your solution in the box close to the printer in Room B 222.