Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 2

Exercise 2.1: *(4 P)*
Use a tableau procedure to prove the satisfiability or unsatisfiability of the following formulae:

(1) $(Q \rightarrow P) \wedge (P \rightarrow Q) \wedge (R \rightarrow Q) \wedge (Q \rightarrow \neg R)$

(2) $(R \rightarrow (P \vee Q)) \wedge (Q \rightarrow (P \wedge R)) \wedge (R \vee Q) \wedge (P \rightarrow \neg R)$

Exercise 2.2: *(1 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be a set of variables containing $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae, which are neither?

(a) $\neg p(g(a), f(x, y))$
(b) $f(x, x) \approx x$
(c) $p(f(x, a), x) \vee p(a, b)$
(d) $p(\neg g(x), g(y))$
(e) $\neg p(f(x, y))$
(f) $p(a, b) \wedge p(x, y) \wedge y$
(g) $\exists y(\neg p(f(y, y), y))$
(h) $\forall x \forall y(g(p(x, y)) \approx g(x))$

Exercise 2.3: *(2 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ h/4,\ a/0,\ b/0\}$ and $\Pi = \{p/2\}$; let X be a set of variables containing $\{x, y, z\}$.
Compute the results of the following substitutions:

(a) $h(a, g(a), g(x), x)[g(a)/x]$
(b) $p(f(y, x), g(x))[x/y]$
(c) $\forall y(p(h(y, x, x, y), g(y)))[x/y]$
(d) $\forall y(p(h(y, x, x, g(y)), x))[y/x]$
(e) $\forall y(p(f(z, g(y)), g(x)) \vee \exists z(h(g(z), y, g(b), b) \approx y))[g(b)/z]$
(f) $\exists y\big(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x)\big)[g(y)/y, g(z)/x]$

Exercise 2.4: *(3 P)*
Let $\Sigma = \{\Omega, \Pi\}$ where $\Omega = \{0, s, +\}$ and $\Pi = \{\approx\}$.
Consider the following formulae in the signature Σ:

1. $F_1 = \forall x \ (x + 0 \approx x)$
2. $F_2 = \forall x, y \ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y \quad (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

> (*Definitions:* A formula F is valid in a Σ-structure $\mathcal{A}$ under assignment β if $\mathcal{A}(\beta)(F) = 1$.
> F is valid in a Σ-structure $\mathcal{A}$ iff $\mathcal{A}, \beta \models F$, for all $\beta \in X \to U_{\mathcal{A}}$.)

Supplementary exercise:

Exercise 2.5: *(2 P)*
Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F.

We will write F also as $F[F']$ in order to emphasize that F' occurs in F. Let $F[P]$ be the formula obtained from F by replacing the subformula F' with the propositional variable P.

Prove:
The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Hint: Use structural induction (see below).

The structural induction principle (for propositional logic).

Let $\mathcal{B}$ be a property of formulae in propositional logic. Assume that the following hold:

- for every propositional variable $P \in \Pi$, P has property $\mathcal{B}$;
- $\bot$ and $\top$ have property $\mathcal{B}$;
- if $F = F_1 \ \mathtt{op} \ F_2$ for $\mathtt{op} \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$ and if both F_1 and F_2 have property $\mathcal{B}$ then F has property $\mathcal{B}$;
- if $F = \neg F_1$ and F_1 has property $\mathcal{B}$ then F has property $\mathcal{B}$.

Then property $\mathcal{B}$ holds for all Π-formulae.

Please submit your solution until Monday, November 5, 2012 in the evening. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibility:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.

- Put your solution in the box close to the printer in Room B 222.