Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 3

Exercise 3.1: (*2 P*)
What is the clausal normal form of

$$\exists x\, ((\ \forall y\, (\forall z\, (p(y,z) \vee \neg\, (x \approx y)))) \rightarrow (\forall z\, q(y,z) \wedge \neg\, r(x,y)))$$

Exercise 3.2: (*1 P*)
Compute a most general unifier of

(1) $\{\, f(x, g(x)) \doteq y,\ h(y) \doteq h(v),\ v \doteq f(g(z), w)\,\}$

(2) $\{\, f(x, g(x)) \doteq y,\ h(y) \doteq h(v),\ v \doteq f(g(x), w)\,\}$

Exercise 3.3: (*2 P*)
Use resolution to show that the following set of clauses is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y)) \ \vee\ p(f(x), y)$$

Exercise 3.4: (*2 P*)
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z, z)$$
$$\neg q(f(x), y) \ \vee\ q(f(f(x)), y) \ \vee\ p(x)$$
$$\neg p(a) \ \vee\ \neg p(f(a)) \ \vee\ q(f(a), f(f(a)))$$
$$p(f(x)) \ \vee\ p(g(y))$$
$$\neg p(g(a)) \ \vee\ p(f(f(a)))$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$.

Exercise 3.5: (*2 P*)
Prove that the following set of formulae is unsafisfiable by using first-order semantic tableaux:

$$\{p(a), \quad \forall x(p(x) \rightarrow p(f(x))), \quad \neg p(f(f(a)))\}$$

Supplementary exercise:

Exercise 3.6: (*2 P*)
Prove or refute the following statements:

(a) If F is a first-order formula, then F is valid if and only if $F \rightarrow \bot$ is unsatisfiable.

(b) If F is a first-order formula and x a variable, then F is unsatisfiable if and only if $\exists x F$ is unsatisfiable.

(c) If F and G are first-order formulae, F is valid, and $F \rightarrow G$ is valid, then G is valid.

(d) If F and G are first-order formulae, F is satisfiable, and $F \rightarrow G$ is satisfiable, then G is satisfiable.

Please submit your solution until Monday, November 5, 2012 in the evening. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibility:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put your solution in the box close to the printer in Room B 222.