Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 4

Exercises 4.1–4.4 refer to the many-valued logic $\mathcal{L}_3$. Let $\Sigma = (\Omega, \Pi)$ be a first-order signature, let $M = \{0, u, 1\}$ and $\mathcal{F}$ be the set of connectives of $\mathcal{L}_3$, and $\mathcal{Q} = \{\forall, \exists\}$.
We will consider first-order formulae over this signature.

Exercise 4.1: *(4 P)*
Prove that the following formulae are $\mathcal{L}_3$ tautologies:

(a) $(\forall x q(x)) \supset q(x)[t/x]$ (for every term t).

(b) $\neg(\exists x F)$ id $\forall x(\neg F)$.

(c) $\sim (\forall x F)$ id $\exists x(\sim F)$.

(d) $\sim (\exists x F)$ id $\forall x(\sim F)$.

where the truth table of id is presented in the slides of lecture 6 (14.11.2011) on page 18.

Exercise 4.2: *(3 P)*
We say that an $\mathcal{L}_3$-formula F is a non-tautology iff for every 3-valued structure, $\mathcal{A}$ and every valuation $\beta : X \to \mathcal{A}$, we have $\mathcal{A}(\beta)(F) \neq 1$. We say that an $\mathcal{L}_3$-formula F is two-valued iff for every 3-valued structure, $\mathcal{A}$ and every valuation $\beta, \mathcal{A}(\beta)(F) \in \{0, 1\}$.

Which of the following statements are true? Justify your answer.

(a) If $F \equiv G$ is a tautology and F is a tautology then G is a tautology.
(b) If F id G is a tautology and F is a tautology then G is a tautology.
(c) If $F \approx G$ is a tautology and F is a tautology then G is a tautology.

(d) If $F \equiv G$ is a tautology and F is satisfiable then G is satisfiable.
(e) If F id G is a tautology and F is satisfiable then G is satisfiable.
(f) If $F \approx G$ is a tautology and F is satisfiable then G is satisfiable.

(g) If $F \equiv G$ is a tautology and F is a non-tautology then G is a non-tautology.
(h) If F id G is a tautology and F is a non-tautology then G is a non-tautology.
(i) If $F \approx G$ is a tautology and F is a non-tautology then G is a non-tautology.

(j) If $F \equiv G$ is a tautology and F is two-valued then G is two-valued.

(k) If $F \ \mathsf{id}\ G$ is a tautology and F is two-valued then G is two-valued.

(l) If $F \approx G$ is a tautology and F is two-valued then G is two-valued.

Exercise 4.3: (*3 P*)
Use the semantic tableau calculus for the many-valued logic $\mathcal{L}_3$ to prove that the following formulae are $\mathcal{L}_3$ tautologies:

(1) $\neg\neg A \ \mathsf{id}\ A$

(2) $\neg(A \vee B) \ \mathsf{id}\ (\neg A \wedge \neg B)$

(3) $\sim (\exists x F) \ \mathsf{id}\ \forall x(\sim F)$

(*Hint:* To avoid the problem of having to use the definition of the operator id the truth table of id can be used for devising suitable expansion rules for the tableau calculus.)

Please submit your solution until Tuesday, November 20, 2012, 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibility:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put your solution in the box close to the printer in Room B 222.