Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 5

The following exercises refer to the many-valued logic $\mathcal{L}_3$. Let $\Sigma = (\Omega, \Pi)$ be a first-order signature, let $M = \{0, u, 1\}$ and $\mathcal{F}$ be the set of connectives of $\mathcal{L}_3$, and $\mathcal{Q} = \{\forall, \exists\}$.

Exercise 5.1: *(3 P)*
Use the semantic tableau calculus for the many-valued logic $\mathcal{L}_3$ to prove that the following formula is an $\mathcal{L}_3$ tautology:

$$\neg(\exists x(p(x) \wedge q(x))) \ \mathsf{id}\ \forall x(\neg(p(x) \wedge q(x))).$$

(*Hint:* To avoid the problem of having to use the definition of the operator id the truth table of id can be used for devising suitable expansion rules for the tableau calculus.)

Exercise 5.2: *(4 P)*
Use the optimized translation to clause form described in the course for computing the CNF for the following signed formulae:

(a) $\{0\}{:}(P \supset Q)$

(b) $\{u\}{:}(P \supset Q)$

(c) $\{1\}{:}(P \supset Q)$

(d) $\{0, 1\}{:}(\sim (P \Rightarrow Q))$

Exercise 5.3: *(3 P)*
Check the unsatisfiability of the set $\{C_1, C_2, C_3\}$ of signed clauses by using the signed propositional resolution calculus.

$$C_1 = P^0$$
$$C_2 = Q^0 \vee Q^u \vee P^1$$
$$C_3 = Q^1 \vee R^u$$
$$C_4 = R^0 \vee R^1$$

(here P^v and $\{v\}{:}P$ are different notations for the same signed literal; similarly for Q and R).

Please submit your solution until Tuesday, December 4, 2012, 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with "Homework Non-Classical Logics" in subject.
- Put your solution in the box close to the printer in Room B 222.