

Exercises for “Non-Classical Logics”
Exercise sheet 7

Exercise 7.1: (3 P)

Use the method presented in the lecture for checking whether the formula

$$F := (P \Rightarrow Q) \vee (Q \Rightarrow P)$$

is a tautology in the Łukasiewicz logic $\mathcal{L}_{\mathbb{N}^1}$, with set of truth values $[0, 1]$. (For checking the satisfiability over $[0, 1]$ of the constraints you obtain use any method you like (e.g. make case distinctions).)

Exercise 7.2: (3 P)

Use the method presented in the lecture for showing that the formula

$$F := (P \vee \neg P)$$

is not a tautology in the Gödel logic with set of truth values $[0, 1]$. (For finding a satisfying assignment into $[0, 1]$ for the constraints you obtain – and thus a variable assignment \mathcal{A} with $\mathcal{A}(F) \neq 1$ – you can use any method you like.)

Exercise 7.3: (2 P)

Consider the inference system for the logic K described in the lecture:

Axioms:

All axioms of propositional logic

$$(K) \quad \Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$$

Inference rules

$$\frac{A \quad A \rightarrow B}{B} \quad [\text{Modus ponens}]$$

$$\frac{A}{\Box A} \quad [G]$$

Prove that $\{Q \rightarrow P, \Box Q\} \vdash \Box(P \wedge Q)$ by constructing a proof of $\Box(P \wedge Q)$ from $N = \{Q \rightarrow P, \Box Q\}$.

Hint: In the proof you can use the following facts:

- in propositional logic $Q \rightarrow P$ is equivalent to $Q \rightarrow (P \wedge Q)$;
- use this equivalence and rule [G] to prove $\Box(Q \rightarrow (P \wedge Q))$ from N ;
- use the axiom schema (K), with $A := Q$ and $B := (P \wedge Q)$;
- apply two times the [Modus ponens] rule.

Please submit your solution until Tuesday, December 18, 2012, 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with “Homework Non-Classical Logics” in subject.
- Put your solution in the box close to the printer in Room B 222.