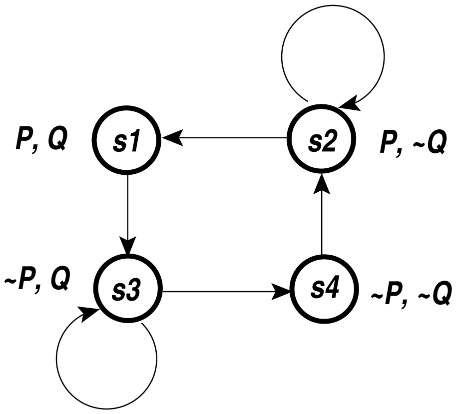
Universität Koblenz-Landau
FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **December 13, 2012**

Exercises for "Non-Classical Logics"
Exercise sheet 8

Exercise 8.1: *(3 P)*
Consider the Kripke model $\mathcal{K} = (S, R, I)$ described below:



(1) Compute:

- $val_{\mathcal{K}}(\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_2)$

(2) Show that at every state $s \in S$ the following hold:

- $val_{\mathcal{K}}(P \wedge Q) \rightarrow \Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((\neg P \wedge \neg Q) \rightarrow \Box P)(s) = 1$
- $val_{\mathcal{K}}((\neg P \wedge \neg Q) \rightarrow \Box\Box P)(s) = 1$

(3) Find a state $s \in S$ such that

$$val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\Box\neg P)(s) = 0.$$

Exercise 8.2: *(2 P)*

(1) Let $\mathcal{K} = (S, R, V)$, and let $s \in S$. Show that if $\{s' \in S \mid sRs'\} = \emptyset$ (i.e. if there is no s' with sRs') then $val_{\mathcal{K}}(\Box F)(s) = 1$ for any formula F.

(2) Find a Kripke frame $\mathcal{F} = (S, R)$ with the property that for every Kripke model $\mathcal{K} = (S, R, I)$, $val_{\mathcal{K}}(\Box \perp)(s) = 1$ for every $s \in S$.

Exercise 8.3: *(3 P)*

(1) Give a property of R that is necessary and sufficient for $\mathcal{F} = (S, R)$ to validate the schema $A \rightarrow \Box A$.

(2) Give a property of R that is necessary and sufficient for $\mathcal{F} = (S, R)$ to validate the schema $\Box \perp$.

(3) Give a property of R that is necessary and sufficient for $\mathcal{F} = (S, R)$ to validate the schema $\Diamond^3 A \rightarrow \Box^2 \Diamond A$.

(4) Can you find an axiom schema which characterized the following property of the relation in a frame: $\forall s_1 \forall s_2 (R^5(s_1, s_2) \rightarrow R^4(s_2, s_1))$

Exercise 8.4: *(4 P)*

Prove that the following formulae are valid (i.e. true in all frames):

(1) $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$

(2) $\Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$

Exercise 8.5: *(3 P)*

Prove that in any Kripke structure $\mathcal{K} = (S, R, I)$ and for every $s \in S$ the following hold:

(1) $(\mathcal{K}, s) \models \Diamond^n F$ if and only if there exists $t \in S$ such that $R^n(s, t)$ and $(\mathcal{K}, t) \models F$.

(2) $(\mathcal{K}, s) \models \Box^n F$ if and only if for all $t \in S$ with $R^n(s, t)$ we have $(\mathcal{K}, t) \models F$.

Exercise 8.6: *(2 P)*

Prove that in any Kripke structure $\mathcal{K}$ the following hold:

(1) If A is a propositional tautology then $\mathcal{K} \models A$.

(2) If $\mathcal{K} \models A$ and $\mathcal{K} \models A \rightarrow B$, then $\mathcal{K} \models B$.

(3) If $\mathcal{K} \models A$ then $\mathcal{K} \models \Box A$.

Please submit your solution until Tuesday, December 18, 2012, 14:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name on your solution.
Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with "Homework Non-Classical Logics" in subject.
- Put your solution in the box close to the printer in Room B 222.