

### Exercises for “Non-Classical Logics” Exercise sheet 1

**Exercise 1.1:** (2 P)

Determine which of the following formulas are valid/satisfiable/unsatisfiable:

- (1)  $(P \wedge Q) \rightarrow P$
- (2)  $Q \wedge \neg Q$
- (3)  $\neg(\neg P \vee \neg\neg P)$
- (4)  $((Q \rightarrow P) \wedge (R \rightarrow \neg P)) \rightarrow (\neg Q \vee \neg R)$

**Exercise 1.2:** (2 P)

Prove Prop. 1.3 (2): If  $N$  is a set of propositional formulas, then  $N \models F$  if and only if  $N \cup \{\neg F\}$  is unsatisfiable.

**Definition:** A set of propositional formulas is unsatisfiable, if and only if for every valuation  $\mathcal{A}$  there is a formula  $G$  in the set such that  $\mathcal{A} \not\models G$  (i.e. if and only if there is no valuation  $\mathcal{A}$  such that  $\mathcal{A} \models G$  for all formulae  $G$  in the set).

**Exercise 1.3:** (3 P)

Let  $F$  be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge \neg P)] \wedge (P \vee R)$$

- (1) Compute the negation normal form (NNF)  $F'$  of  $F$ .
- (2) Convert  $F'$  to CNF using:
  - (a) distributivity of disjunctions over conjunctions?
  - (b) the satisfiability-preserving transformation described in the lecture.

**Exercise 1.4:** (2 P)

Consider the formulae  $F_n = \bigvee_{i=1}^n (Q_i \wedge R_i)$  for  $n \in \mathbb{N}$ .

As a function of  $n$ , how many clauses are in:

- (1) the CNF formula  $F'$  constructed using the distributivity of disjunctions over conjunctions?
- (2) the CNF formula  $F''$  obtained using the satisfiability-preserving translation to clause form?
- (3) For which  $n$  is the first approach better?

**Exercise 1.5:** (2 P)

Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

- (1)  $\neg P \vee \neg Q \vee R$
- (2)  $\neg P \vee \neg Q \vee S$
- (3)  $P$
- (4)  $\neg S \vee \neg R$
- (5)  $Q$

**Exercise 1.6:** (2 P)

Assume  $S \succ P \succ Q \succ R$ . Let  $N$  be the following set of clauses:

- (1)  $\neg Q \vee \neg P$
- (2)  $R \vee P$
- (3)  $Q \vee S$
- (4)  $\neg Q \vee \neg S$

How are the clauses in  $N$  ordered w.r.t. the multiset extension of  $\succ$ ?

Please submit your solution until Wednesday, October 30, 2013, at 10:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.