Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

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Exercises for "Non-Classical Logics"
Exercise sheet 12

Exercise 12.1: *(4 P)*
Consider the formula $F = \Box Q \vee Q$. Check the satisfiability of the formula using the following steps:

- Construct the set of clauses N corresponding to $\exists x P_F(x) \wedge \mathsf{Rename}(F)$
- Use the ordered resolution with selection calculus $\mathsf{Res}_S^{\succ}$ introduced in the lecture for checking the satisfiability of N.

Exercise 12.2: *(4 P)*
Snomed CT is a comprehensive clinical healthcare terminology that comprises more than 400 000 vocabulary items and almost the same number of logical axioms. An example of concept which is defined in Snomed CT is apendicitis.

Assume that we have:
– the concept names: Disease, Appendicitis, Inflammation, Appendix and
– the role names: AssociatedMorphology, FindingSite.

(1) An example of a typical Snomed CT axiom mentions that appendicits is a disease which is morphologically an (= some) inflammation. In ALC this can be expressed as follows:

$$\mathsf{Appendicitis} \sqsubseteq \mathsf{Disease} \sqcap \exists \mathsf{AssociatedMorphology}.\mathsf{Inflammation}$$

How can this be expressed in first-order logic?

(2) How can one express in ALC the fact that appendicitis is a disease which is located in the (= some) appendix?

Supplementary exercises
(to be discussed in the next exercise class)

Exercise 12.3: (*10 P*)
Let F be a formula in propositional modal logic, F' a subformula of F, and F'' another formula.

F' has positive polarity in F if it occurs under an even number of negations (we think of $A \to B$ as $\neg A \vee B$). Otherwise, F' has negative polarity in F.

Prove:

(1) Assume F' has positive polarity in F. Let $\mathcal{K} = (S, R, I)$.
If $(\mathcal{K}, s) \models F[F']$ and for all $t \in S$ we have $(\mathcal{K}, t) \models (F' \to F'')$ then $(\mathcal{K}, s) \models F[F'']$.

(2) Assume F' has negative polarity in F. Let $\mathcal{K} = (S, R, I)$.
If $(\mathcal{K}, s) \models F[F']$ and for all $t \in S$ we have $(\mathcal{K}, t) \models (F'' \to F')$ then $(\mathcal{K}, s) \models F[F'']$.

Exercise 12.4: (*5 P*)
Let F be a formula in propositional modal logic, and F' a subformula of F. Let P be a new propositional variable, not occurring in F.

(3) Assume F' has positive polarity in F.
Then $F[F']$ is satisfiable iff there exists a Kripke model $\mathcal{K} = (S, R, I)$ and $s \in S$ such that $(\mathcal{K}, s) \models F[P]$ and for every state $t \in S$ we have $(\mathcal{K}, t) \models (P \to F')$.

(4) Assume F' has negative polarity in F.
Then $F[F']$ is satisfiable iff there exists a Kripke model $\mathcal{K} = (S, R, I)$ and $s \in S$ such that $(\mathcal{K}, s) \models F[P]$ and for every state $t \in S$ we have $(\mathcal{K}, t) \models (F' \to P)$.

Please submit your solution until Tuesday, January 28, 2014 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.