Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **January 29, 2014**

Exercises for "Non-Classical Logics"
Exercise sheet 13

Exercise 13.1: (*6 P*)
We suppose that in the set of all possible objects there is a set of objects that are trees and a binary relation direct-subtree between objects that leads from a tree to its direct subtrees.

Then the binary trees are the trees with at most two direct subtrees and such that all these direct subtrees are themselves binary trees.

(1) Write a formal description of the concept of binary tree in SHIQ. Indicate the set of concepts and roles which you used for this.

(2) Assume that we additionally consider the proper-subtree relation, with the following properties:

- Every direct subtree of a tree T is a proper subtree of T.
- If T_1 is a proper subtree of T_2 and T_2 is a proper subtree of T_3 then T_1 is a proper subtree of T_3 (transitivity).

Present a SHIQ formalism in which you can also consider this relation, by specifying:

- a set N_C of concept names;
- a set N_R^0 of atomic role symbols which you need for the specification;
- a subset $N_t^0 \subseteq N_R^0$ of transitive role symbols needed for the specification;
- a hierarchy on roles.

Is the description of the concept of a binary tree you gave in (1) a correct SHIQ concept description over this extended language? Justify your answer.

Exercise 13.2: (*4 P*)
Show that the following formulae are valid in propositional dynamic logic (i.e. true in all PDL Kripke models) :

(1) $[\alpha](A \wedge B) \leftrightarrow [\alpha]A \wedge [\alpha]B$
(2) $[\alpha; \beta]A \leftrightarrow [\alpha][\beta]A$
(3) $[\alpha \cup \beta]A \leftrightarrow [\alpha]A \wedge [\beta]A$
(4) $[A?]B \leftrightarrow (A \rightarrow B)$

(5) $[\alpha^*]A \leftrightarrow A \wedge [\alpha][\alpha^*]A$

(6) $[\alpha^*](A \rightarrow [\alpha]A) \rightarrow (A \rightarrow [\alpha^*]A)$

Supplementary exercises
(suggested by Manuel Mittler,
to be discussed during the exercise class)

Exercise 13.3: (* P)
Check using the tableau calculus whether the following subsumption relations hold:

(1) $B \sqcap \forall R.\neg B \quad \sqsubseteq \quad B \sqcap \forall R.(\neg B \sqcup \forall R.\neg B)$

(2) $B \sqcap \forall R.(\neg B \sqcup \forall R.\neg B) \quad \sqsubseteq \quad B \sqcap \forall R.\neg B$

Exercise 13.4: (** P)
Consider the following TBox $\mathcal{T}$:

$A \sqsubseteq B \sqcap \exists R.\neg A$

$B \sqsubseteq \neg C \sqcap \forall R.E.$

Check using the tableau calculus whether $\mathcal{T} \models A \sqsubseteq \neg C \sqcap \forall R.(E \sqcup A)$.

Please submit your solution until Tuesday, February 4, 2014 at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.