Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **November 8, 2013**

Exercises for "Non-Classical Logics"
Exercise sheet 3

Exercise 3.1: (*3 P*)
Let $\Sigma = \{\Omega, \Pi\}$ where $\Omega = \{0, s, +\}$ and $\Pi = \{\approx\}$.
Consider the following formulae in the signature Σ:

1. $F_1 = \forall x\ (x + 0 \approx x)$
2. $F_2 = \forall x, y\ (x + s(y) \approx s(x + y))$
3. $F_3 = \forall x, y\ \ (x + y \approx y + x)$.

Find a Σ-structure in which F_1 and F_2 are valid but F_3 is not.

(*Definitions:* A formula F is valid in a Σ-structure $\mathcal{A}$ under assignment β if $\mathcal{A}(\beta)(F) = 1$.
F is valid in a Σ-structure $\mathcal{A}$ iff $\mathcal{A}, \beta \models F,$ for all $\beta \in X \to U_{\mathcal{A}}$.)

Exercise 3.2: (*2 P*)
What is the clausal normal form of

$$\exists x\,((\ \forall y\,(\forall z\,(p(y,z) \vee \neg\,(x \approx y)))) \to (\forall z\,q(y,z) \wedge \neg\,r(x,y)))$$

Exercise 3.3: (*1 P*)
Compute a most general unifier of

(1) $\{\, f(x, g(x)) \doteq y,\ h(y) \doteq h(v),\ v \doteq f(g(z), w)\,\}$
(2) $\{\, f(x, g(x)) \doteq y,\ h(y) \doteq h(v),\ v \doteq f(g(x), w)\,\}$

Exercise 3.4: (*2 P*)
Use resolution to show that the following set of clauses is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y))\ \vee\ p(f(x), y)$$

Exercise 3.5: (*2 P*)
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\begin{gathered}
\neg q(z,z) \\
\neg q(f(x),y) \quad \vee \quad q(f(f(x)),y) \quad \vee \quad p(x) \\
\neg p(a) \quad \vee \quad \neg p(f(a)) \quad \vee \quad q(f(a),f(f(a))) \\
p(f(x)) \quad \vee \quad p(g(y)) \\
\neg p(g(a)) \quad \vee \quad p(f(f(a)))
\end{gathered}$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res^{\succ}_{S}$.

Exercise 3.6: (*2 P*)
Prove that the following set of formulae is unsafisfiable by using first-order semantic tableaux:

$$\{p(a), \quad \forall x(p(x) \rightarrow p(f(x))), \quad \neg p(f(f(a)))\}$$

Please submit your solution until Tuesday, November 12, 2013, at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.