Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **November 13, 2013**

Exercises for "Non-Classical Logics"
Exercise sheet 4

Exercise 4.1: (*2 P*)
Use the resolution calculus Res to show that the following set of clauses is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y)) \ \vee \ p(f(x), y)$$

Exercise 4.2: (*2 P*)
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z, z)$$
$$\neg q(f(x), y) \ \vee \ q(f(f(x)), y) \ \vee \ p(x)$$
$$\neg p(a) \ \vee \ \neg p(f(a)) \ \vee \ q(f(a), f(f(a)))$$
$$p(f(x)) \ \vee \ p(g(y))$$
$$\neg p(g(a)) \ \vee \ p(f(f(a)))$$

(a) Which literals are maximal in the clauses of N?
(b) Define a selection function S such that N is saturated under $Res_S^{\succ}$.

Exercise 4.3: (*2 P*)
Prove that the following set of formulae is unsafisfiable by using first-order semantic tableaux (with free variables):

$$\Big(\ \exists x \ \forall y \big(p(x, y) \wedge q(x)\big)\Big) \ \wedge \ \Big(\neg\big(\exists x\big(p(x, f(x))\big) \wedge \exists x \ q(x)\big) \ \Big)$$

Exercise 4.4: (*2 P*)
Let Π be a set of propositional variables, let $M = \{0, u, 1\}$ and $\mathcal{F} = \{\vee/2, \wedge/2, \neg/1, \sim/1\}$ be the set of connectives of the propositional version of the many-valued logic $\mathcal{L}_3$ presented in the lecture.

Let F be a formula in the propositional logic $\mathcal{L}_3$ with propositional variables in Π. Show that if the formula F does not contain $\sim$ then it cannot be an $\mathcal{L}_3$ tautology.

Please submit your solution until Tuesday, November 19, 2013, at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.