Universität Koblenz-Landau

FB 4 Informatik

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November 20, 2013

Exercises for "Non-Classical Logics"
Exercise sheet 5

Exercise 5.1: (*2 P*)
Prove that the following formulae are $\mathcal{L}_3$ tautologies:

(a) $(\forall x q(x)) \supset q(x)[t/x]$ (for every term t).

(b) $\neg(\exists x F)$ id $\forall x(\neg F)$.

(c) $\sim (\forall x F)$ id $\exists x(\sim F)$.

(d) $\sim (\exists x F)$ id $\forall x(\sim F)$.

where the truth table of id is presented in the slides of lecture from 20.11.2013.

Exercise 5.2: (*3 P*)
We say that an $\mathcal{L}_3$-formula F is a non-tautology iff for every 3-valued structure, $\mathcal{A}$ and every valuation $\beta : X \to \mathcal{A}$, we have $\mathcal{A}(\beta)(F) \neq 1$. We say that an $\mathcal{L}_3$-formula F is two-valued iff for every 3-valued structure, $\mathcal{A}$ and every valuation $\beta, \mathcal{A}(\beta)(F) \in \{0, 1\}$.

Which of the following statements are true? Justify your answer.

(a) If $F \equiv G$ is a tautology and F is a tautology then G is a tautology.
(b) If F id G is a tautology and F is a tautology then G is a tautology.
(c) If $F \approx G$ is a tautology and F is a tautology then G is a tautology.

(d) If $F \equiv G$ is a tautology and F is satisfiable then G is satisfiable.
(e) If F id G is a tautology and F is satisfiable then G is satisfiable.
(f) If $F \approx G$ is a tautology and F is satisfiable then G is satisfiable.

(g) If $F \equiv G$ is a tautology and F is a non-tautology then G is a non-tautology.
(h) If F id G is a tautology and F is a non-tautology then G is a non-tautology.
(i) If $F \approx G$ is a tautology and F is a non-tautology then G is a non-tautology.

(j) If $F \equiv G$ is a tautology and F is two-valued then G is two-valued.
(k) If F id G is a tautology and F is two-valued then G is two-valued.

(l) If $F \approx G$ is a tautology and F is two-valued then G is two-valued.

Exercise 5.3: *(3 P)*
Use the semantic tableau calculus for the many-valued logic $\mathcal{L}_3$ to prove that the following formulae are $\mathcal{L}_3$ tautologies:

(1) $\neg\neg A \ \mathsf{id}\ A$

(2) $\neg(A \vee B) \ \mathsf{id}\ (\neg A \wedge \neg B)$

(3) $\sim (\exists x F) \ \mathsf{id}\ \forall x(\sim F)$

(*Hint:* To avoid the problem of having to use the definition of the operator id the truth table of id can be used for devising suitable expansion rules for the tableau calculus.)

Please submit your solution until Tuesday, November 26, 2013, at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework Non-Classical Logics” in the subject.
- Put it in the box in front of Room B 222.