Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **December 11, 2013**

Exercises for "Non-Classical Logics"
Exercise sheet 8

Exercise 8.1: *(2 P)*
Consider the inference system for the logic K described in the lecture:

Axioms:

All axioms of propositional logic

$(K) \quad \Box(A \to B) \to (\Box A \to \Box B)$

Inference rules

$$\frac{A \quad A \to B}{B} \quad \text{[Modus ponens]}$$

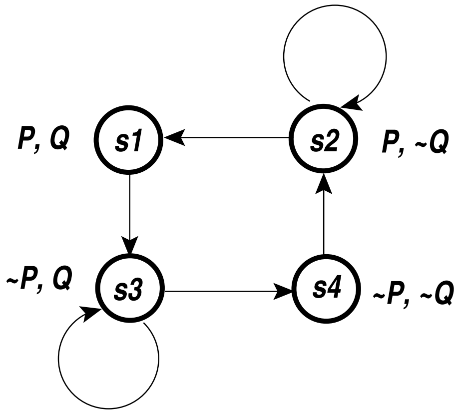
$$\frac{A}{\Box A} \quad \text{[G]}$$

Prove that $\{Q \to P, \Box Q\} \vdash \Box(P \wedge Q)$ by constructing a proof of $\Box(P \wedge Q)$ from $N = \{Q \to P, \Box Q\}$.

Hint: In the proof you can use the following facts:

- in propositional logic $Q \to P$ is equivalent to $Q \to (P \wedge Q)$;
- use this equivalence and rule [G] to prove $\Box(Q \to (P \wedge Q))$ from N;
- use the axiom schema (K), with $A := Q$ and $B := (P \wedge Q)$;
- apply two times the [Modus ponens] rule.

Exercise 8.2: *(3 P)*
Consider the Kripke model $\mathcal{K} = (S, R, I)$ described below:



(1) Compute:

- $val_{\mathcal{K}}(\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_2)$

(2) Show that at every state $s \in S$ the following hold:

- $val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((\neg P \wedge \neg Q) \rightarrow \Box P)(s) = 1$
- $val_{\mathcal{K}}((\neg P \wedge \neg Q) \rightarrow \Box\Box P)(s) = 1$

(3) Find a state $s \in S$ such that

$$val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\Box\neg P)(s) = 0.$$

Exercise 8.3: *(2 P)*

(1) Let $\mathcal{K} = (S, R, V)$, and let $s \in S$. Show that if $\{s' \in S \mid sRs'\} = \emptyset$ (i.e. if there is no s' with sRs') then $val_{\mathcal{K}}(\Box F)(s) = 1$ for any formula F.

(2) Find a Kripke frame $\mathcal{F} = (S, R)$ with the property that for every Kripke model $\mathcal{K} = (S, R, I)$, $val_{\mathcal{K}}(\Box \perp)(s) = 1$ for every $s \in S$.

Supplementary exercise:

Remember the “wise men” and “muddy children” examples given in the lecture (at the beginning of the section on modal logic).

We will formalize the “wise men” example and show how to derive information about the knowledge of the wise men in the corresponding inference system.

Please submit your solution until Tuesday, December 17, 2013, at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword “Homework Non-Classical Logics” in the subject.
- Put it in the box in front of Room B 222.