

### Exercises for “Non-Classical Logics” Exercise sheet 9

#### Exercise 9.1: (2 P)

- (1) Give a property of  $R$  that is necessary and sufficient for  $\mathcal{F} = (S, R)$  to validate the schema  $A \rightarrow \Box A$ .
- (2) Give a property of  $R$  that is necessary and sufficient for  $\mathcal{F} = (S, R)$  to validate the schema  $\Diamond^3 A \rightarrow \Box^2 \Diamond A$ .
- (3) Can you find an axiom schema which characterized the following property of the relation in a frame:  $\forall s_1 \forall s_2 (R^5(s_1, s_2) \rightarrow R^4(s_2, s_1))$

#### Exercise 9.2: (4 P)

Prove that the following formulae are valid (i.e. true in all frames):

- (1)  $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$
- (2)  $\Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$

#### Exercise 9.3: (3 P)

Prove that in any Kripke structure  $\mathcal{K} = (S, R, I)$  and for every  $s \in S$  the following hold:

- (1)  $(\mathcal{K}, s) \models \Diamond^n F$  if and only if there exists  $t \in S$  such that  $R^n(s, t)$  and  $(\mathcal{K}, t) \models F$ .
- (2)  $(\mathcal{K}, s) \models \Box^n F$  if and only if for all  $t \in S$  with  $R^n(s, t)$  we have  $(\mathcal{K}, t) \models F$ .

#### Exercise 9.4: (2 P)

Prove that in any Kripke structure  $\mathcal{K}$  the following hold:

- (1) If  $A$  is a propositional tautology then  $\mathcal{K} \models A$ .
- (2) If  $\mathcal{K} \models A$  and  $\mathcal{K} \models A \rightarrow B$ , then  $\mathcal{K} \models B$ .
- (3) If  $\mathcal{K} \models A$  then  $\mathcal{K} \models \Box A$ .

Please submit your solution until Tuesday, January 7, 2014, at 16:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword “Homework Non-Classical Logics” in the subject.
- Put it in the box in front of Room B 222.