

Exercises for “Non-Classical Logics”

Exercise sheet 1

Exercise 1.1: (2 P)

Determine which of the following formulas are valid/satisfiable/unsatisfiable:

- (1) $(P \wedge Q) \rightarrow P$
- (2) $Q \wedge \neg Q$
- (3) $\neg(\neg P \vee \neg\neg P)$
- (4) $((Q \rightarrow P) \wedge (R \rightarrow \neg P)) \rightarrow (\neg Q \vee \neg R)$

Exercise 1.2: (2 P)

Prove Prop. 1.3 (2): If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable.

Definition: A set of propositional formulas is unsatisfiable, if and only if for every valuation \mathcal{A} there is a formula G in the set such that $\mathcal{A} \not\models G$ (i.e. if and only if there is no valuation \mathcal{A} such that $\mathcal{A} \models G$ for all formulae G in the set).

Exercise 1.3: (2 P)

Determine the polarity of the following subformulae of

$$F = \neg((\neg(P \wedge \neg Q)) \vee (R \vee \neg S)) \vee (U \wedge V)$$

- (1) $(P \wedge \neg Q)$
- (2) Q
- (3) $(R \vee \neg S)$
- (4) S
- (5) V
- (6) $((\neg(P \wedge \neg Q)) \vee (R \vee \neg S))$

Exercise 1.4: (3 P)

Let F be the following formula:

$$\neg[((Q \wedge \neg P) \wedge \neg(Q \wedge R)) \rightarrow (Q \wedge \neg P)] \wedge (P \vee R)$$

- (1) Compute the negation normal form (NNF) F' of F .
- (2) Convert F' to CNF using:
 - (a) distributivity of disjunctions over conjunctions.
 - (b) the satisfiability-preserving transformation described in the lecture.

Exercise 1.5: (2 P)

Consider the formulae $F_n = \bigvee_{i=1}^n (Q_i \wedge R_i)$ for $n \in \mathbb{N}$.

As a function of n , how many clauses are in:

- (1) the CNF formula F' constructed using the distributivity of disjunctions over conjunctions?
- (2) the CNF formula F'' obtained using the satisfiability-preserving translation to clause form?
- (3) For which n is the first approach better?

Supplementary exercises

(to be discussed in one of the following exercise sessions)

Exercise 1.6: (2 P)

Let F be a formula, P a propositional variable not occurring in F , and F' a subformula of F . Prove: The formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Exercise 1.7: (4 P)

Let F be a formula containing neither \rightarrow nor \leftrightarrow , P a propositional variable not occurring in F , and F' a subformula of F . Prove:

- If F' has positive polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (P \rightarrow F')$ is satisfiable.
- If F' has negative polarity in F then $F[F']$ is satisfiable if and only if $F[P] \wedge (F' \rightarrow P)$ is satisfiable.

Please submit your solution until Monday, November 3, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.