Universität Koblenz-Landau

FB 4 Informatik

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Exercises for "Non-Classical Logics"
Exercise sheet 11

Exercise 11.1: *(2 P)*
Compute the translation into first order logic used for checking the validity of a modal formula Φ (of the form $\exists x P_{\neg\Phi}(x) \wedge \mathsf{Rename}(\neg\Phi)$) for the following formulae:

(1) $\Phi_1 : \quad \Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$
(2) $\Phi_3 : \quad ((\Box\Diamond P \wedge \Diamond P) \rightarrow \Diamond\Box P)$

Exercise 11.2: *(4 P)*
Consider the formula $F = \Box Q \vee Q$. Check the satisfiability of the formula using the following steps:

- Construct the set of clauses N corresponding to $\exists x P_F(x) \wedge \mathsf{Rename}(F)$
- Use the ordered resolution with selection calculus $\mathsf{Res}_S^{\succ}$ introduced in the lecture for checking the satisfiability of N.

Supplementary exercises
(to be discussed in the next exercise class)

Exercise 11.3: *(10 P)*
Let F be a formula in propositional modal logic, F' a subformula of F, and F'' another formula.

F' has positive polarity in F if it occurs under an even number of negations (we think of $A \rightarrow B$ as $\neg A \vee B$). Otherwise, F' has negative polarity in F.

Prove:

(1) Assume F' has positive polarity in F. Let $\mathcal{K} = (S, R, I)$.
If $(\mathcal{K}, s) \models F[F']$ and for all $t \in S$ we have $(\mathcal{K}, t) \models (F' \rightarrow F'')$ then $(\mathcal{K}, s) \models F[F'']$.

(2) Assume F' has negative polarity in F. Let $\mathcal{K} = (S, R, I)$.
If $(\mathcal{K}, s) \models F[F']$ and for all $t \in S$ we have $(\mathcal{K}, t) \models (F'' \rightarrow F')$ then $(\mathcal{K}, s) \models F[F'']$.

Exercise 11.4: *(5 P)*
Let F be a formula in propositional modal logic, and F' a subformula of F. Let P be a new propositional variable, not occurring in F.

(3) Assume F' has positive polarity in F.
Then $F[F']$ is satisfiable iff there exists a Kripke model $\mathcal{K} = (S, R, I)$ and $s \in S$ such that $(\mathcal{K}, s) \models F[P]$ and for every state $t \in S$ we have $(\mathcal{K}, t) \models (P \rightarrow F')$.

(4) Assume F' has negative polarity in F.
Then $F[F']$ is satisfiable iff there exists a Kripke model $\mathcal{K} = (S, R, I)$ and $s \in S$ such that $(\mathcal{K}, s) \models F[P]$ and for every state $t \in S$ we have $(\mathcal{K}, t) \models (F' \rightarrow P)$.

Please submit your solution until Monday, February 2, 2015, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.