Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **November 5, 2014**

Exercises for "Non-Classical Logics"
Exercise sheet 2

Exercise 2.1: (*2 P*)
Let F be the following formula:

$$\neg[((Q \wedge \neg P) \vee \neg(Q \vee R)) \rightarrow ((Q \rightarrow P) \wedge (Q \wedge \neg P))] \wedge (P \vee R)$$

(1) Compute the negation normal form (NNF) F' of F.

(2) Convert F' to CNF using the satisfiability-preserving transformation described in the lecture.

Exercise 2.2: (*3 P*)
Use the resolution calculus to prove that the following set of clauses is unsatisfiable:

(1)	$\neg P \vee \neg Q \vee R$
(2)	$\neg P \vee \neg Q \vee S$
(3)	P
(4)	$\neg S \vee \neg R$
(5)	Q

Exercise 2.3: (*2 P*)
Assume $S \succ P \succ Q \succ R$. Let N be the following set of clauses:

(1)	$\neg Q \vee \neg P$
(2)	$R \vee P$
(3)	$Q \vee S$
(4)	$\neg Q \vee \neg S$

(1) Which literals are maximal in the clauses of N?

(2) Which inferences are possible in the ordered resolution calculus $\mathsf{Res}^{\succ}$ with the rules:

$$\frac{C \vee A \qquad D \vee \neg A}{C \vee D} \quad \text{[ordered resolution]}$$

if C, D are clauses and A is a propositional variable with:
(i) $A \succ C$ (A is larger (in $\succ$) than the maximal literal in C);
(ii) $\neg A \succeq \max(D)$ (i.e. $\neg A$ is larger than or equal to the maximal literal of D).

$$\frac{C \vee A \vee A}{(C \vee A)} \qquad \text{[ordered factoring]}$$

if C is a clause and A a propositional variable such that A is maximal in C.

- Let S be the selection function which selects the negative literal $\neg Q$ in the clauses (1) and (4). Which inferences are possible in the ordered resolution calculus with selection $\mathsf{Res}_S^{\succ}$ presented in the lecture on slides 36 and 37.

Supplementary exercise:

Exercise 2.4: *(4 P)*
Let F be a formula, P a propositional variable not occurring in F, and F' a subformula of F.

We denote by $F[F']$ the formula F (in which F' is not replaced) and by $F[P]$ the formula obtained from F by replacing the subformula F' with the propositional variable P.

(1) Prove that for any valuation $\mathcal{A}$, if $\mathcal{A}(P) = \mathcal{A}(F')$ then $\mathcal{A}(F[P]) = \mathcal{A}(F[F'])$

(2) Use (1) to show that $F[P] \wedge (P \leftrightarrow F')$ is satisfiable if and only if $F[F']$ is satisfiable.

Please submit your solution until Monday, November 10, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.