

Exercises for “Non-Classical Logics” Exercise sheet 3

Exercise 3.1: (4 P)

Let N be the following set of clauses:

- (1) $\neg P_3 \vee P_1 \vee P_1$
- (2) $\neg P_2 \vee P_1$
- (3) $P_4 \vee P_4$
- (4) P_4
- (5) $P_3 \vee \neg P_2$
- (6) $P_4 \vee P_3$

- (1) Let \succ be the ordering on propositional variables defined by $P_4 \succ P_3 \succ P_2 \succ P_1$. Sort the clauses in N according to \succ_C . Which literals are maximal in the clauses of N ?
- (2) Define a selection function S such that N is saturated under $Res_S^<$.
- (3) Construct a model of N using the canonical construction presented in the lecture.

Exercise 3.2: (4 P)

Use a tableau procedure to prove the satisfiability or unsatisfiability of the following formulae:

- (1) $(Q \rightarrow P) \wedge (P \rightarrow Q) \wedge (R \rightarrow Q) \wedge (Q \rightarrow \neg R)$
- (2) $(R \rightarrow (P \vee Q)) \wedge (Q \rightarrow (P \wedge R)) \wedge (R \vee Q) \wedge (P \rightarrow \neg R)$

Exercise 3.3: (1 P)

Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2, g/1, a/0, b/0\}$ and $\Pi = \{p/2\}$; let X be a set of variables containing $\{x, y, z\}$. Which of the following expressions are terms over Σ and X , which are atoms/literals/clauses/formulae, which are neither?

- (a) $\neg p(g(a), f(x, y))$
- (b) $f(x, x) \approx x$
- (c) $p(f(x, a), x) \vee p(a, b)$
- (d) $p(\neg g(x), g(y))$
- (e) $\neg p(f(x, y))$
- (f) $p(a, b) \wedge p(x, y) \wedge y$
- (g) $\exists y(\neg p(f(y, y), y))$
- (h) $\forall x \forall y(g(p(x, y)) \approx g(x))$

Exercise 3.4: (1 P)

Consider the structure \mathbb{N} introduced in the lecture from 12.11.2014 (on slides 46 and 47). Let $x, y \in X$ and let $\beta : X \rightarrow U_{\mathbb{N}}$ with $\beta(x) = 1$ and $\beta(y) = 3$.

Compute $\mathbb{N}(\beta)(\forall x \exists y (x < y))$.

Please submit your solution until Monday, November 17, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to sofronie@uni-koblenz.de with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.