

### Exercises for “Non-Classical Logics” Exercise sheet 4

#### Exercise 4.1: (3 P)

Let  $\Sigma = \{\Omega, \Pi\}$  where  $\Omega = \{0, s, +\}$  and  $\Pi = \{\approx\}$ .

Consider the following formulae in the signature  $\Sigma$ :

1.  $F_1 = \forall x (x + 0 \approx x)$
2.  $F_2 = \forall x, y (x + s(y) \approx s(x + y))$
3.  $F_3 = \forall x, y (x + y \approx y + x)$ .

Find a  $\Sigma$ -structure in which  $F_1$  and  $F_2$  are valid but  $F_3$  is not.

#### Exercise 4.2: (2 P)

Compute a clausal normal form for the formula:

$$\exists x ((\forall y (\forall z (p(y, z) \vee \neg(x \approx y)))) \rightarrow (\forall z q(y, z) \wedge \neg r(x, y)))$$

#### Exercise 4.3: (1 P)

Compute a most general unifier of

- (1)  $\{f(x, g(x)) \doteq y, h(y) \doteq h(v), v \doteq f(g(z), w)\}$
- (2)  $\{f(x, g(x)) \doteq y, h(y) \doteq h(v), v \doteq f(g(x), w)\}$

#### Exercise 4.4: (2 P)

Use the resolution calculus Res to show that the following set of clauses is unsatisfiable:

$$\begin{aligned} & p(a, z) \\ & \neg p(f(f(a)), a) \\ & \neg p(x, g(y)) \vee p(f(x), y) \end{aligned}$$

#### Exercise 4.5: (2 P)

Let  $\succ$  be a total and well-founded ordering on ground atoms such that, if the atom  $A$  contains more symbols than  $B$ , then  $A \succ B$ . Let  $N$  be the following set of clauses:

$$\begin{array}{c}
\neg q(z, z) \\
\neg q(f(x), y) \vee q(f(f(x)), y) \vee p(x) \\
\neg p(a) \vee \neg p(f(a)) \vee q(f(a), f(f(a))) \\
p(f(x)) \vee p(g(y)) \\
\neg p(g(a)) \vee p(f(f(a)))
\end{array}$$

- (a) Which literals are maximal in the clauses of  $N$ ?
- (b) Define a selection function  $S$  such that  $N$  is saturated under  $Res_{\zeta}^>$ .

**Exercise 4.6:** (2 P)

Prove that the following set of formulae is unsatisfiable by using first-order semantic tableaux (with free variables):

$$\left( \exists x \forall y (p(x, y) \wedge q(x)) \right) \wedge \left( \neg (\exists x (p(x, f(x))) \wedge \exists x q(x)) \right)$$

Please submit your solution until Monday, November 24, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.