

### Exercises for “Non-Classical Logics” Exercise sheet 6

#### Exercise 6.1: (4 P)

Let  $g : \{0, u, 1\} \times \{0, u, 1\} \rightarrow \{0, u, 1\}$  be the function defined as described in the table:

$g$		0	$u$	1
0		1	0	$u$
$u$		0	$u$	0
1		$u$	0	1

Construct a formula  $F$  with propositional variables  $P_1, P_2$  in the (functionally complete) propositional logic  $\mathcal{L}_3^+$  defined in the lecture from 3.12.2014 which “has the same truth table as  $g$ ”, i.e. has the property that for every  $\mathcal{A} : \{P_1, P_2\} \rightarrow \{0, u, 1\}$ :

$$g(\mathcal{A}(P_1), \mathcal{A}(P_2)) = \mathcal{A}(F).$$

#### Exercise 6.2: (6 P)

Use the semantic tableau calculus for the many-valued logic  $\mathcal{L}_3$  to prove that the following formulae are  $\mathcal{L}_3$  tautologies:

- (1)  $\neg\neg A$  id  $A$
- (2)  $\neg(A \vee B)$  id  $(\neg A \wedge \neg B)$
- (3)  $\sim (\exists x F(x))$  id  $\forall x(\sim F(x))$

(*Hint:* To avoid the problem of having to use the definition of the operator id the truth table of id can be used for devising suitable expansion rules for the tableau calculus.)

Please submit your solution until Monday, December 8, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to [sofronie@uni-koblenz.de](mailto:sofronie@uni-koblenz.de) with the keyword “Homework Non-Classical Logics” in the subject.
- Put it in the box in front of Room B 222.