Universität Koblenz-Landau

FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans **December 17, 2014**

Exercises for "Non-Classical Logics"
Exercise sheet 8

Exercise 8.1: (*3 P*)
Use the method presented in the lecture for checking whether the formula

$$F := (P \Rightarrow Q) \vee (Q \Rightarrow P)$$

is a tautology in the Łukasiewicz logic $\mathcal{L}_{\aleph_1}$, with set of truth values $[0, 1]$. (For checking the satisfiability over $[0, 1]$ of the constraints you obtain use any method you like (e.g. make case distinctions).)

Exercise 8.2: (*3 P*)
Use the method presented in the lecture for showing that the formula

$$F := (P \vee \neg P)$$

is not a tautology in the Gödel logic with set of truth values $[0, 1]$. (For finding a satisfying assignment into $[0, 1]$ for the constraints you obtain – and thus a variable assignment $\mathcal{A}$ with $\mathcal{A}(F) \neq 1$ – you can use any method you like.)

Exercise 8.3: (*2 P*)
Consider the inference system for the logic K described in the lecture:

Axioms:

All axioms of propositional logic

(K) $\quad \Box(A \to B) \to (\Box A \to \Box B)$

Inference rules

$$\frac{A \quad A \to B}{B} \quad \text{[Modus ponens]}$$

$$\frac{A}{\Box A} \quad \text{[G]}$$

Prove that $\{Q \to P, \Box Q\} \vdash \Box(P \wedge Q)$ by constructing a proof of $\Box(P \wedge Q)$ from $N = \{Q \to P, \Box Q\}$.

Hint: In the proof you can use the following facts:

- in propositional logic $Q \to P$ is equivalent to $Q \to (P \wedge Q)$;

- use this equivalence and rule [G] to prove $\Box(Q \rightarrow (P \wedge Q))$ from N;
- use the axiom schema (K), with $A := Q$ and $B := (P \wedge Q)$;
- apply two times the [Modus ponens] rule.

Please submit your solution until Monday, January 12, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.