Universität Koblenz-Landau

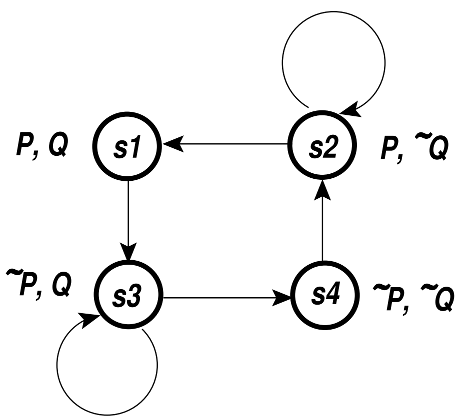
FB 4 Informatik

Prof. Dr. Viorica Sofronie-Stokkermans

January 14, 2015

Exercises for "Non-Classical Logics"
Exercise sheet 9

Exercise 9.1: (*3 P*)
Consider the Kripke model $\mathcal{K} = (S, R, I)$ described below:



By labelling a possible world s with the propositional variable P resp. Q we indicate that $I(P, s) = 1$, resp. $I(Q, s) = 1$; by labelling it with $\sim P$ ($\sim Q$) we indicate that $I(P, s) = 0$ (resp. $I(Q, s) = 0$).

(1) Compute:

- $val_{\mathcal{K}}(\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_1)$
- $val_{\mathcal{K}}(\Box\Box\neg P)(s_2)$

(2) Show that at every state $s \in S$ the following hold:

- $val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\neg P)(s) = 1$
- $val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\neg P)(s) = 1$

(3) Find a state $s \in S$ such that

$$val_{\mathcal{K}}((P \wedge Q) \rightarrow \Box\Box\Box\neg P)(s) = 0.$$

Exercise 9.2: (*2 P*)

(1) Let $\mathcal{K} = (S, R, V)$, and let $s \in S$. Show that if $\{s' \in S \mid sRs'\} = \emptyset$ (i.e. if there is no s' with sRs') then $val_{\mathcal{K}}(\Box F)(s) = 1$ for any formula F.

(2) Find a Kripke frame $\mathcal{F} = (S, R)$ with the property that for every Kripke model $\mathcal{K} = (S, R, I)$, $val_{\mathcal{K}}(\Box \perp)(s) = 1$ for every $s \in S$.

Exercise 9.3: (*2 P*)

(1) Give a property of R that is necessary and sufficient for $\mathcal{F} = (S, R)$ to validate the schema $A \rightarrow \Box A$.

(2) Give a property of R that is necessary and sufficient for $\mathcal{F} = (S, R)$ to validate the schema $\Diamond^3 A \rightarrow \Box^2 \Diamond A$.

(3) Can you find an axiom schema which characterized the following property of the relation in a frame: $\forall s_1 \forall s_2 (R^5(s_1, s_2) \rightarrow R^4(s_2, s_1))$

Exercise 9.4: (*4 P*)
Prove that the following formulae are valid (i.e. true in all frames):

(1) $\Diamond(P \vee Q) \leftrightarrow (\Diamond P \vee \Diamond Q)$

(2) $\Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$

Exercise 9.5: (*3 P*)
Prove that in any Kripke structure $\mathcal{K} = (S, R, I)$ and for every $s \in S$ the following hold:

(1) $(\mathcal{K}, s) \models \Diamond^n F$ if and only if there exists $t \in S$ such that $R^n(s, t)$ and $(\mathcal{K}, t) \models F$.

(2) $(\mathcal{K}, s) \models \Box^n F$ if and only if for all $t \in S$ with $R^n(s, t)$ we have $(\mathcal{K}, t) \models F$.

Please submit your solution until Monday, January 19, 2014, at 18:00. Joint solutions prepared by up to three persons are allowed. Please do not forget to write your name(s) on your solution.

Submission possibilities:

- By e-mail to `sofronie@uni-koblenz.de` with the keyword "Homework Non-Classical Logics" in the subject.
- Put it in the box in front of Room B 222.