# Non-Classical Logics

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# Organization

## Organization

3h Lecture + 1h Exercises

### Time:

Wednesday: Lecture/Exercise 10:00 c.t.-12:00, Room C 208

Wednesday: Lecture: 16:00 c.t.-18:00, Room E 016

discuss possibilities of changing the time 16:00-18:00

**website:** `http://www.uni-koblenz.de/ sofronie/lecture-non-classical-ws-2014/`

### Homework

- will be avaliable online after the lecture on Wednesday
(at latest on Thursday evening); due on next Monday at 17:00.

# Exam

## Exam:

– form (oral/written): to be decided

# Non-Classical Logics

- **Alternatives to classical logic**
- **Extensions of classical logic**

# Non-Classical Logics

- **Alternatives to classical logic**

  Accept or reject certain theorems of classical logic following intuitions arising from significant application areas and/or from human reasoning.

# Non-Classical Logics

- **Alternatives to classical logic**

  Examples:

  - many-valued logics
  - intuitionistic logic
  - substructural logics
    (accept only some of the structural rules of classical logic)
  - partial logics
    (sentences do not have to be either true or false; terms do not have to be always defined)
  - free logics
    (agree with classical logic at propositional level; differ at the predicate logic level)
  - quantum logics
    (connection with problems in physical systems)

# Non-Classical Logics

- **Extensions of classical logic**

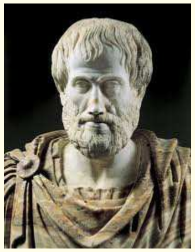
  Extensions of classical logic by means of new operators

  - modal logic
  - dynamic logic
  - temporal logic

# Motivation and History

The nature of logic and knowledge has been studied and debated since ancient times.

**Aristotle**



Traditionally, in Aristotle's logical calculus, there were only two possible values (i.e., "true" and "false") for any proposition.

He noticed however, that there are sentences (e.g. referring to future events) about which it is difficult to say whether they are true or false, although they can be either true or false (De Interpretatione, ch. IX).

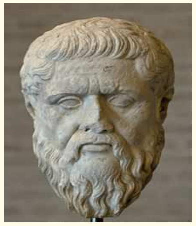
Example: "Tomorrow there will be a naval battle."

Aristotle didn't create a system of non-classical logic to explain this isolated remark.

# Motivation and History

The nature of logic and knowledge has been studied and debated since ancient times.

### Platon



Platon postulated that there is a third “area” between

the notions of true and false.

“knowledge is always proportionate to the realm from which it is gained

deterministic school/non-deterministic school

Until the 20th century logicians mainly followed Aristotelian logic, which includes or assumes the law of the excluded middle.

# Motivation and History

**John Duns Scotus** (1266 - 1308)

Reasoned informally in a modal manner, mainly to analyze statements about possibility and necessity.

**William of Ockham** (1288 - 1348)

Wrote down in words the formulae that would later be called De Morgan's Laws, and pondered ternary logic, that is, a logical system with three truth values (distinguishing "neutral" propositions from true and false ones) a concept that would be taken up again in the mathematical logic of the 20th century.

# Motivation and History

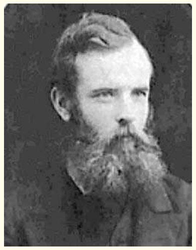
- **George Boole** (1815 - 1864)

| | |
|---|---|
| 1847 | Mathematical Analysis of Logic |
| 1854 | An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities |

Boole's approach founded what was first known as the "algebra of logic" tradition.
$\mapsto$ Boolean algbra (classical logic!)

# Motivation and History

- **Hugh Mac Coll** (1837 - 1909)



  - first known variant of the propositional calculus, which he called "calculus of equivalent statements"
  - Explored the possibilities of modal logic, logic of fiction, connexive logic, many-valued logic and probability logic.

- **Charles Sanders Peirce** (1839 - 1914)

  - Important contributions to logic and its understanding.
    - NAND/NOR
    - predicate logic
  - Introduces e.g. logic of relatives, relational logic (further developed by Tarski)

# History and Motivation

In the 20th century, a systematic study of non-classical logics started.

In a tentative of avoiding logical paradoxes in 1939 **Bochvar** adds one more truth value ("meaningless")

> Idea: e.g. in Russell's paradox, declare the crucial sentences involved as meaningless:
>
> $R = \{x \mid \neg(x \in x)\}$ $\qquad$ $R \in R$ iff $\neg(R \in R)$
>
> declare "$R \in R$" as meaningless.

# History and Motivation

Many-valued logics were introduced to model undefined or vague information:

- **Jan Łukasiewicz** began to create systems of many-valued logic in 1920, using a third value "possible" to deal with Aristotle's paradox of the sea battle.
- **Emil L. Post** (1921) introduced the formulation of additional truth degrees with $n \geq 2$ where $n$ is the number of truth values (starting mainly from algebraic considerations).
- Later, **Jan Łukasiewicz** and **Alfred Tarski** together formulated a logic on $n$ truth values where $n \geq 2$.
- **Stephen Cole Kleene** introduced a 3-valued logic in order to express the fact that some recursive functions might be undefined.
- In 1932 **Hans Reichenbach** formulated a logic of many truth values where $n = \infty$.

# History and Motivation

Many-valued logics were introduced to model undefined or vague information:

- **Paul Bernays** (1926) used 3-valued logics for proving the independence of a given axiomatic system for classical propositional logic.

  (this way of proving independence requires a high degree of creativity, since for each special case a suitable many-valued logic must be found)

Fuzzy logics; probabilistic logic

- **Lotfi Zadeh** (1965) developed the theory of fuzzy sets which led to the study of fuzzy logic.
- **Nils Nilsson** (1986) proposes a logic where the truth values of sentences are probabilities (probabilistic logic).

# History and Motivation

## Constructive mathematics

| | |
|---|---|
| $A$ true: | there exists a proof for $A$ |
| $A \vee B$ true: | there exists a proof for $A$ or there exists a proof for $B$ |
| | hence: $A \vee \neg A$ is not always true; $A \leftrightarrow \neg\neg A$ is not always true |
| $\exists x P(x)$ true: | there exists $x_0$ that can be constructed effectively, and there exists a proof that $P(x_0)$ is true. |

$\mapsto$ **Intuitionistic Logic**

- **Luitzen Egbertus Jan Brouwer** (1907-1908)
- **V. Glivenko** (fragment of propositional logic)
- **A.N. Kolmogorov** (fragment of predicate logic)
- **Arend Heyting** (1928, 1930)
  Heyting gave the first formal development of intuitionistic logic in order to codify Brouwer's way of doing mathematics.

# History and Motivation

Kurt Gödel (in 1932) showed that intuitionistic logic is not a finitely-many valued logic, and defined a system of Gödel logics intermediate between classical and intuitionistic logic; such logics are known as intermediate logics.

# History and Motivation

### Alternatives to classical logics

Study properties of implication, logical entailment or premise combination.

- Relevant logic
  $X \vdash A$ holds: $X$ must be relevant for $A$

  It may happen that $X \vdash A$ holds and $X, Y \vdash A$ does not hold.

- Linear logic
  Premises are seen as resources which must be used and cannot be reused.

- Lambek calculus
  Premise combination: combination of linguistic units
  (both the number and the order of the premises are important)

# History and Motivation

**Extensions of classical logic** by means of new logical operators

## Modal logic

- modal operators $\Box$, $\Diamond$

| meaning of $\Box A$ | meaning of $\Diamond A$ |
|---|---|
| $A$ is necessarily true | $A$ is possibly true |
| An agent believes $A$ | An agent thinks $A$ is possible |
| $A$ is always true | $A$ is sometimes true |
| $A$ should be the case | $A$ is allowed |
| $A$ is provable | $A$ is not contradictory |

# History and Motivation

## Logics related to modal logic

### Dynamic logic of programs

Operators:

$[\alpha]A$: $A$ holds after every run of the (non-deterministic) process $\alpha$

$\langle\alpha\rangle A$: $A$ holds after some run of the (non-deterministic) process $\alpha$

# History and Motivation

## Logics related to modal logic

### Temporal logic

| | |
|---|---|
| $\Box A$: | $A$ holds always (in the future) |
| $\Diamond A$: | $A$ holds at some point (in the future) |
| $\bigcirc A$: | $A$ holds at the next time point (in the future) |
| $A$ until $B$ | $A$ must remain true at all following time points until $B$ becomes true |

# History and Motivation

**Extensions of classical logic: Modal logic and related logics**

Very rich history:

- Antiquity and middle ages (**John Duns Scotus**, **Willian of Ockham**)
- **C. I. Lewis** founded modern modal logic in his 1910 Harvard thesis.
- **Ruth C. Barcan** (later Ruth Barcan Marcus) developed the first axiomatic systems of quantified modal logic.
- In 1959, **Saul Kripke** (then a 19-year-old Harvard student) introduced the possible-worlds semantics for modal logics.
- **A. N. Prior** created modern temporal logic in 1957
- **Vaughan Pratt** introduced dynamic logic in 1976.
- In 1977, **Amir Pnueli** proposed using temporal logic to formalise the behaviour of continually operating concurrent programs.

# Structure of this course

- Classical logic (reminder)
- Many-valued logic
- Modal logic and related logics
  (e.g. dynamic logic and description logics)
- Temporal logic

# Classical logic

- Propositional logic (Syntax, Semantics)
- First-order logic (Syntax, Semantics)

Proof methods (resolution, tableaux)

# Many-valued logic

- Introduction
- Many-valued logics

    3-valued logic

    finitely-valued logic

    fuzzy logic

- Reduction to classical logic

# Modal logic

- Introduction, history
- Introduction to propositional modal logic

  Syntax and semantics

  Correspondence theory

  Completeness, canonical models

  Decidability
- Introduction to first-order modal logics
- Reduction to first-order logic
- Description logics
- Dynamic logic

# Temporal logic

- Linear temporal logic
- Branching temporal logic
- Model checking

# Literature

Peter Schmitt's lecture notes on non-classical logics
(in German, linked from the website of the lecture)

# Literature

## Additional literature:

### Modal, temporal and dynamic logic


- Bull and Segerberg Basic modal logic. In Handbook of Philosophical Logic,
- Fitting, M. Basic modal logic. In Handbook of Logic in Artificial Intelligence and Logic Programming, Vol 1: Logical Foundations. 368-448
- Fitting, M. Proof methods for modal and intuitionistic logics, Kluwer, 1983.
- Fitting, M. and Mendelsohn, R. First-order modal logic, Kluwer, 1998
- Goldblatt, R. Logics of time and computation, CSLI Series, 1987
- Hughes, G.E. and Cresswell, M.J.
  - A new introduction to modal logic, 1st ed., Routledge, 1996.
  - A companion to modal logic, Methuen, 1985.
  - Introduction to modal logic (repr. 1990), Routledge, 1972.
- Huth, M. and Ryan, M. Logic in Computer Science: Modelling and reasoning about systems, Cambridge University Press, 2000

# Literature

## Additional literature:

### Modal and temporal logic


- Stirling, C. Modal and temporal logics. In Handbook of Logics in Computer Science, Vol 2: Background: Computational Structures (Gabbay, D. and Abramski, S. and Maibaum, T.S.E. eds), pages 478-563, Clarendon Press, 1992.
- Stirling, C. Modal and temporal properties of processes, Springer Texts in computer science, 2001.
- Emerson, E.A. Temporal and modal logic. Handbook of Theoretical Computer Science, 1990.
- Kroeger, F. Temporal logic of programs, EATCS monographs on theoretical computer science, Springer, 1987.
- Clarke, E.N., Emerson, E.A., Sistla, A.P.: Automatic verification of finite-state concurrent systems using temporal logic specifications. ACM Transactions on Programming Languages and Systems (TOPLAS) 8(2): 244-263

# Literature

**Additional literature:**

## Modal and dynamic logic

- Harel, D., Kozen, D. and Tiuryn, J. Dynamic logic, MIT Press, 2000

**Semesterapparat at the library**