Non-classical logics

Lecture 5: Many-valued logics

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Until now

Classical logic

- Propositional logic (Syntax, Semantics)
- First-order logic (Syntax, Semantics)

Proof methods (resolution, tableaux)

From now on: Non-Classical logics

- Many-valued logic (finitely-valued; infinitely-valued)

  Syntax, semantics, Automated proof methods (resolution, tableaux)

  Reduction to classical logic

- Modal logics (also description logics, dynamic logic)

  Syntax, semantics, Automated proof methods (resolution, tableaux)

  Reduction to classical logic

- Temporal logic (Linear time; branching time)

  Syntax, semantics, Model checking

From now on: Non-Classical logics

- Many-valued logic (finitely-valued; infinitely-valued)

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- Modal logics (also description logics, dynamic logic)

  Syntax, semantics, Automated proof methods (resolution, tableaux)

  Reduction to classical logic

- Temporal logic (Linear time; branching time)

  Syntax, semantics, Model checking

Many-valued logic

- Introduction
- Many-valued logics

  3-valued logic

  finitely-valued logic

  fuzzy logic
- Automated theorem proving (resolution, tableaux)
- Reduction to classical logic

History and Motivation

Many-valued logics were introduced to model undefined or vague information

History and Motivation

Jan Łukasiewicz
Began to create systems of many-valued logic in 1920, using a third value "possible" to deal with Aristotle's paradox of the sea battle.

- Jan Łukasiewicz:
  "On 3-valued logic" (Polish) Ruch Filozoficzny, Vol. 5, 1920.

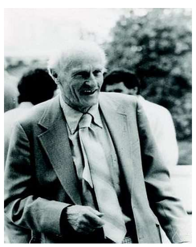
Later, **Jan Łukasiewicz** and **Alfred Tarski** together formulated a logic on n truth values where $n \geq 2$.

- Jan Łukasiewicz:
  Philosophische Bemerkungen zu mehrwertigen Systemen des Aussagenkalküls. Comptes rendus des séance de la Societé des Sciences et des Lettres de Varsovie, Classe III, Vol .23, 1930.
- S. McCall:
  Polish Logic: 1920–1939. Oxford University Press, 1967.

History

Emile L. Post
Introduced (in 1921) the formulation of additional truth degrees with $n \geq 2$ where n is the number of truth values (starting mainly from algebraic considerations).

- Emil Post:
  Introduction to a general theory of elementary propositions. American J. of Math., Vol. 43, 1921.



S. C. Kleene:

Introduced a 3-valued logic in order to express the fact that some recursive functions might be undefined.

Applications of many-valued logic

- independence proofs
- modeling undefined function and predicate values (program verification)
- semantic of natural languages
- theory of logic programming: declarative description of operational semantics of negation
- modeling of electronic circuits
- modeling vagueness and uncertainly
- shape analysis (program verification)

Literature

- J. B. Rosser, A. R. Turquette: Many-valued Logics. North-Holland, 1952.
- N. Rescher: Many-valued Logic. McGraw-Hill, 1989.
- Alasdair Urquhart: Handbook of Philosophical Logic, vol. 3, 1986.
- Bolc und Borowik: Many-Valued Logics. Springer Verlag 1992,

Literature

- Matthias Baaz, Christian G. Fermüller:
  Resolution-Based Theorem Proving for Many valued Logics.
  J. Symb. Comput. 19(4): 353-391 (1995)
- Reiner Hähnle:
  Automated Deduction in Multiple-valued Logics.
  Clarendon Press, Oxford, 1993.
- Grzegorz Malinowski:
  Many-Valued Logics.
  Oxford Logic Guides, Vol. 25, Clarendon Press, Oxford, 1993.
- Siegfried Gottwald
  A Treatise On Many-Valued Logics. Studies in Logic and Computation,
  Vol. 9, Research Studies Press, 2001.

Literature

- Harald Ganzinger and Viorica Sofronie-Stokkermans
  Chaining techniques for automated theorem proving in many-valued logic. ISMVL 2000.

- Viorica Sofronie-Stokkermans and Carsten Ihlemann
  Automated reasoning in some local extensions of ordered structures
  Multiple-Valued Logic and Soft Computing 13(4-6): 397-414, 2007.

A motivating example

B: the sky is blue

R: it rains

U: I take my umbrella

$$(B \to \neg R) \land (R \to U) \land (B \to \neg U) \land R$$

A motivating example

B: the sky is blue

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$$(B \rightarrow \neg R) \wedge (R \rightarrow U) \wedge (B \rightarrow \neg U) \wedge R$$

Description of a situation: (partial) variable assignment $v : \Pi \rightarrow \{0, 1\}$

A	$v(A)$
B	1
R	
U	0

Truth tables in partial logic

v partial valuation.

$v \sqsubseteq v_1$: v_1 is a total variable assignment which extends v.

Example

Description of a situation:

(partial) $v : \Pi \rightarrow \{0, 1\}$

A	$v(A)$
B	1
R	
U	0

A	$v_1(A)$	$v_2(A)$
B	1	1
R	0	1
U	0	0

$v \sqsubseteq v_1$, $v \sqsubseteq v_2$

$v(F_1 \wedge F_2) = 0$ iff for all v_1 with $v \sqsubseteq v_1$ we have $v_1(F_1 \wedge F_2) = 0$

$v(F_1 \wedge F_2) = 1$ iff for all v_1 with $v \sqsubseteq v_1$ we have $v_1(F_1 \wedge F_2) = 1$

Truth tables for partial logic

$\wedge$	1	undef	0
1	1	undef	0
undef	undef	undef	0
0	0	0	0

$\vee$	1	undef	0
1	1	1	1
undef	1	undef	undef
0	1	undef	0

F	$\neg F$
1	0
undef	undef
0	1

A motivating example

$\wedge$	1	undef	0
1	1	undef	0
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$\vee$	1	undef	0
1	1	1	1
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F	$\neg F$
1	0
undef	undef
0	1

$$(B \rightarrow \neg R) \wedge (R \rightarrow U) \wedge (B \rightarrow \neg U) \wedge R$$

Description of a situation: (partial) variable assignment $v : \Pi \rightarrow \{0, 1\}$

A	$v(A)$
B	1
R	undef
U	0

F	$v(F)$
$\neg B \vee \neg R$	undef
$\neg R \vee U$	undef
$\neg B \vee \neg U$	1

Another example

Belnap's 4-valued logic

This particularly interesting system of MVL was the result of research on relevance logic, but it also has significance for computer science applications. Its truth degree set may be taken as

$$M = \{\{\}, \{0\}, \{1\}, \{0, 1\}\},$$

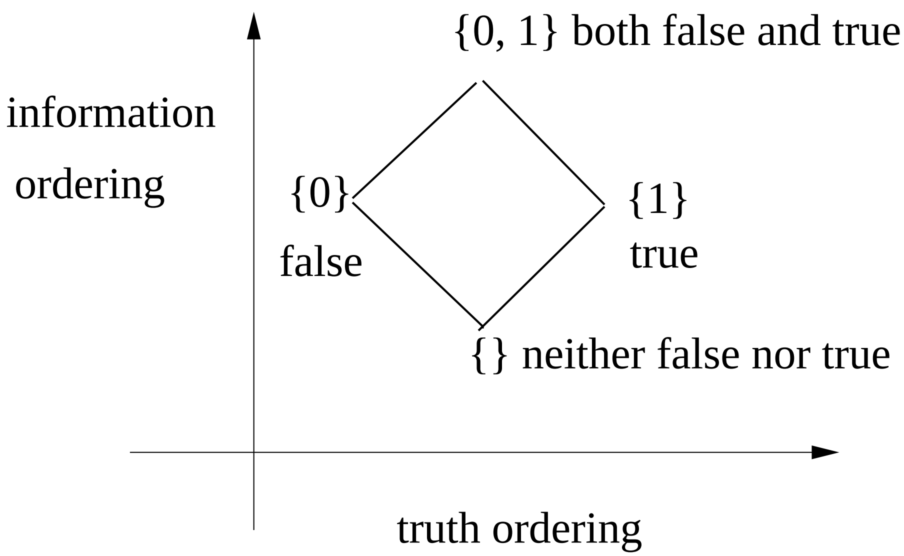
and the truth degrees interpreted as indicating (e.g. with respect to a database query for some particular state of affairs) that there is

- no information concerning this state of affairs,
- information saying that the state of affairs is false,
- information saying that the state of affairs is true,
- conflicting information saying that the state of affairs is true as well as false.

Another example

Belnap's 4-valued logic $\quad M = \{\{\}, \{0\}, \{1\}, \{0,1\}\}$

This set of truth degrees has two natural orderings:



$\wedge, \vee$: sup/inf in the truth ordering

$\sim \{\} = \{\}, \quad \sim \{0,1\} = \{0,1\}, \quad \sim \{0\} = \{1\}, \quad \sim \{1\} = \{0\}$

"Designated" values: (What we can assume to be true)

Computer science: $D = \{\{1\}\}$

Other applications (e.g. information bases): $D = \{\{1\}, \{0,1\}\}$