# Non-classical logics
# Summary of topics

1. **Classical logic**
   - **Propositional logic**
     - Syntax; semantics; models, validity, satisfiability, entailment, equivalence
     - Translation to CNF/DNF (in particular structure-preserving translations!)
     - Resolution; semantic tableaux
   - **First-order logic**
     - Syntax, semantics: models and assignments; validity, satisfiability; entailment and equivalence
     - Validity vs. unsatisfiability
     - Normal forms and Skolemization
     - General resolution (form of inference rules); unification (definition of a most general unifier; algorithm for computing a most general unifier) (No proofs required)
     - Semantic tableaux (No proofs required)
2. **Many-valued logics**
   - Propositional finitely valued logics: Syntax and semantics; models validity, satisfiability
   - First-order finitely valued logics: Syntax and semantics (interpretation of quantifiers!) models validity, satisfiability;
   - The many-valued logic $\mathcal{L}_3$ (you should know the truth tables of $\wedge, \vee, \sim, \neg$)
   - Functional completeness:
     - definition
     - criterion for functional completeness (criterion; idea of the proof)
     - examples of many-valued logics which are not functionally complete ($\mathcal{L}_3$, why?)
     - examples of functionally complete many-valued logics (classical logic, $\mathcal{L}_3^+$ (idea), Post logics (idea))

- Examples of many-valued logics (Post logics, Łukasiewics logics)
- Proof calculi
  - general proof calculi and inference systems (provability; soundness/completeness/refutational completeness: definitions)
  - proof calculus for $\mathcal{L}_3$ (you do not have to learn the inference rules by hard, but you should be able to use it for proving $\mathcal{L}_3$ theorems)
- Automated reasoning:
  - signed formulae; tableaux rules for finitely valued logics
  - translation to signed CNF; signed resolution: the two forms presented on the slides.
- Application in logic and verification (not required for the exam)

3. **Infinitely-valued logics**
   - Łukasiewicz logics (finitely valued; infinitely valued): Definitions; Main properties:
     - Link between tautologies of $\mathcal{L}_n$ and $\mathcal{L}_m$ if $(m-1)|(n-1)$ and the converse of this theorem (statement; idea of proof)
     - Relationship between the tautologies of $\mathcal{L}_{\aleph_0}$ and those of $\mathcal{L}_{\aleph_1}$
     - Relationship between the tautologies of $\mathcal{L}_{\aleph_0}$ and the tautologies of $\mathcal{L}_n$, $n \in \mathbb{N}$
   - "Fuzzy" logics: t-norms; definition; examples (Łukasiewicz logic, Gödel logic, Product logic)
   - Checking validity of formulae in fuzzy logics: reduction to checking constraints over $[0, 1]$ (Idea; applying the algorithm)

4. **Modal logic**
   - Motivation
   - Syntax
     - Signature; formulae
     - Proof theory (inference systems - general definitions; soundness, completeness)
     - Inference system for modal logics; proofs
   - The modal system $K$ (you do not need to learn the axioms of the other systems of modal logic, but you should be able to use them for proving theorems in the modal logic $K$)
   - Semantics:
     - Kripke frames and Kripke structures
     - Models, validity, satisfiability
     - Entailment (global entailment; local entailment; the deduction theorem)

- Correspondence theory
  - Correspondence theorem: the general result linking the property $C(m, n, j, k)$ of $R$ and axiom $\Diamond^m \Box^n P \rightarrow \Box^j \Diamond^k P$
  - Application: correspondence theorems for the properties/axiom schemata in the list given on the slides.
- First-order definability
  - Irreflexivity does not correspond to the validity of any modal schema (idea of proof)
- Theorem proving (for the modal logic $K$):
  - Axiom system (soundness; completeness: idea of the proof – construction of canonical models)
  - semantic tableaux (soundness and completeness: only idea)
  - translation to first-order logic (+ resolution)
- Decidability of the modal logic $K$ (idea of the proof – construction of filtrations; properties of filtrations; finite model property + computable bound on size of models; sketch of decision procedure; extensions to other modal logics)

5. **Description logics:** ALC (definition; link with the modal logic $K$); SHIQ (definition)

6. **Dynamic logic** (main definitions; completeness and finite model property: main idea of the proof)